Recalling that "number" is actually $(x - 3)$, you get

$$(x - 3) \times 2x + (x - 3) \times 5.$$

Distributing the $2x$ and the 5 gives

$$2x^2 - 6x + 5x - 15,$$

which can be simplified by combining like terms to give

$$2x^2 - x - 15.$$

Once you understand the procedure, you can multiply two binomials *quickly*, in your head. Just multiply each term of one binomial by each term of the other, and write down the answer.

The following exercise is designed to give you practice identifying and naming polynomials, and multiplying binomials.

EXERCISE 1-4

Do These Quickly

The following problems are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. Evaluate: $23 - 3 \cdot 7$

Q2. Evaluate $|12x - 4|$ if x is -2.

Q3. Distribute the 0.5: $0.5(8x - 9t)$

Q4. Find 70% of 700.

Q5. Subtract and simplify: $\frac{2}{9} - \frac{8}{9}$

Q6. What axiom was used? $(3 + x) + 4 = (x + 3) + 4$

Q7. What axiom tells that 2 times 3 is a unique real number?

Q8. Is $\sqrt{48}$ a rational number?

Q9. Square 7.

Q10. Write an expression representing 17 more than x.

For Problems 1 through 24, tell whether or not the given expression is a polynomial. If it *is*, then name it according to its *degree* and number of *terms* (e.g., "quadratic trinomial"). If it is *not* a polynomial, then tell *why* not.

1. $3x^4 - 2x + 51$

2. $3^2x^3 + 5y^4$

3. $9xy + 2z$
4. $9xyz + 2$
5. $3x^2y - \frac{57}{x}$
6. $3x^2y - \sqrt{57x}$
7. $\frac{x}{2} - 1$
8. $\sqrt{x} \times 5$
9. $\frac{2}{x} - 1$
10. $x + \sqrt{5}$
11. -13
12. 19
13. $8x^3 + 5x^2 - 2x + 11$
14. $3x^2 + 5x - 7$
15. $x^3y^2 - \pi$
16. $xy^5 - \pi^7$
17. $\sqrt{x} + 11$
18. $\frac{x}{4} - 17$
19. $x + \sqrt{11}$
20. $\frac{4}{x} - 17$
21. $5^2x^3y^7 + 8z^9$
22. $6^3x^5y^2 - 3z^6$
23. 0
24. $0x^5$

For Problems 25 through 34, multiply the two binomials.

25. $(x - 3)(x + 7)$
26. $(x - 6)(x + 5)$
27. $(x + 4)(2x - 1)$
28. $(3x + 1)(x - 2)$
29. $(3x - 8)(2x - 7)$
30. $(4x - 3)(7x - 5)$
31. $(2x - 5)(2x - 5)$
32. $(3x - 10)(3x - 10)$
33. $(2x - 5)^2$
34. $(3x - 10)^2$

1-5 EQUATIONS

Evaluating an expression may be thought of as finding out what number the expression equals when you know the value of the variable. You are ready to *reverse* the process, and find out the value of the *variable* when you know what number the *expression* equals. For example, if

$$3x - 5 = 16,$$

then x must equal 7, since $3 \times 7 - 5$ equals 16. The statement "$3x - 5 = 16$" is an equation, and the process of writing down what x

must equal is called *solving* the equation. Since there may be more than one solution, it is customary to write the solutions in a *solution set*. For the above equation,

$$S = \{7\}.$$

AGREEMENT

Solving an equation means writing its solution set.

Most of the equations you will encounter are too complicated to solve by inspection. So the procedure is to transform them to one or more equations of the form

$$x = \text{a constant}.$$

Then the solution set *can* be written by inspection.

EXAMPLE 1

Solve $3x - 5 = 16$.

Solution:
The technique is to transform the equation to $x =$ constant by getting rid of the unwanted numbers around the x in the left member.

$3x - 5 = 16$	Write the given equation.
$3x = 21$	Add 5 to each member.
$x = 7$	Multiply each member by $\frac{1}{3}$.

This equation is *equivalent* to the original one, meaning it has the *same* solution set. By inspection, you can write

$$S = \{7\}.$$

Notes:

1. Before writing the solution set, you should substitute the value(s) of x back into the original equation to be sure you have made no mistakes.
2. You should not stop at the step "$x = 7$." Since you have agreed that solving an equation means writing the solution set, the equation is not solved until you write "$S = \{7\}$."
3. Adding (or subtracting) and multiplying (or dividing) both members of an equation by the same number are justified by the *Addition Property of Equality* and the *Multiplication Property of Equality*, respectively.

These properties can be proved from the axioms, as you will see in Section 1-7.

EXAMPLE 2

Solve $x + 3 = x$.

Solution:
This equation has no solutions at all, since no number comes out the same when 3 is added to it. The solution set is *empty*, and you would write

$$S = \emptyset \text{ or } S = \{\ \}.$$

EXAMPLE 3

Solve $(3x - 4)(x + 5) = 0$.

Solution:
This equation contains a *product* which equals *zero*. The only way a product of two real numbers can equal zero is for one of the factors to equal zero. This fact is expressed in the converse of the *Multiplication Property of Zero,* which can be proved from the axioms as in Section 1-7.

Because of this fact, you can transform the equation to

$$3x - 4 = 0 \text{ or } x + 5 = 0.$$

Further transformations give

$3x = 4$ or $x = -5$	Adding 4 or subtracting 5.
$x = \frac{4}{3}$ or $x = -5$	Dividing by 3.
$\therefore S = \{\frac{4}{3}, -5\}$.	Solving the transformed equation.

Note: In order for a number to be a solution, it must be in the *domain* of the variable. If the domain of x were

$$x \in \{\text{positive numbers}\},$$

then $S = \{\frac{4}{3}\}$. If the domain is

$$x \in \{\text{integers}\},$$

then $S = \{-5\}$. If the domain is

$$x \in \{\text{natural numbers}\},$$

then $S = \emptyset$, since neither of the possible solutions is a positive integer.

EXAMPLE 4

Solve $|x - 2| = 3$.

Solution:
By inspection you might see that x could equal 5. But x could also equal -1. To make sure you do not overlook any solutions, you try to transform the equation to equivalent equations of the form $x =$ a constant. You can

do this by realizing that there are just *two* numbers whose absolute value is 3, namely 3 and −3. So the expression inside the absolute value sign, $x - 2$, must equal one of these two numbers. You would write

$$x - 2 = 3 \text{ or } x - 2 = -3.$$

Adding 2 to each member of each equation gives

$$x = 5 \text{ or } x = -1,$$

so that

$$S = \{5, -1\}.$$

Extraneous Solutions and Irreversible Steps — Two equations are *equivalent* if they have the *same* solution set. Unfortunately, there are transformations which you can perform on an equation which seem perfectly correct, but which *change* the solution set. Since it is the *original* equation you are trying to solve, you must be aware of the types of transformations which might *not* produce an equivalent equation.

1. *Multiply by an expression which can equal zero* — For example, the equation

$$x = 5$$

has $S = \{5\}$. But if you multiply both members by $x - 2$ you get

$$x(x - 2) = 5(x - 2).$$

This transformed equation is still true when $x = 5$, as the Multiplication Property of Equality tells you it must be. But it is also true when $x = 2$ because

$$2(2 - 2) = 5(2 - 2) \quad \text{or} \quad 2 \times 0 = 5 \times 0$$

is also a true statement. So 2 is a solution of the transformed equation which *does not* satisfy the original equation. Such a solution is called an *extraneous* solution, "extra-" meaning "added," and "-neouis" meaning "new."

DEFINITION

An **extraneous** solution is a number which satisfies a transformed equation, but not the original equation.

Mutiplying both members of an equation by an expression which can equal zero is called an *irreversible step*. You cannot go backwards and divide both members by that expression since division by zero is undefined. Irreversible steps sometimes produce extraneous solutions.

2. *Divide by an expression that can equal zero* — For example, the equation

$$x^2 = 4x$$

has the solution set $S = \{4, 0\}$, as you can see by substituting these numbers. But if you divide both members by x you get

$$x = 4,$$

which has only $S = \{4\}$. So dividing by a variable can *lose* a valid solution.

The following exercise is designed to give you practice solving equations of the type you should recall from previous mathematics courses. You will also demonstrate that you understand about transformations that do *not* produce equivalent equations.

EXERCISE 1-5

Do These Quickly

The following problems are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. Find 40% of 700.

Q2. What axiom expresses the fact that 0 + 1776 equals 1776?

Q3. What degree is 5^2x^7y?

Q4. $x^2 + 7x - 2$ is a ____nomial. What goes in the blank?

Q5. Is π an irrational number?

Q6. Add and simplify: $\frac{2}{3} + \frac{6}{7}$

Q7. Square -5.

Q8. Distribute: $3(x^2 - 5x + 11)$

Q9. Is $\frac{5x}{13}$ a polynomial?

Q10. Is 0.666666... (repeating) a rational number?

For Problems 1 through 22, solve the equation in the indicated domain.

1. $3x + 7 = -8$ {real numbers}

2. $4x - 6 = 10$ {real numbers}
3. $2x + 3 = x - 1$ {positive numbers}
4. $5x - 1 = 2x + 5$ {negative numbers}
5. $5x + 3 = 2x + 3$ {real numbers}
6. $5x + 3 = 5x - 4$ {real numbers}
7. a. $4x + 3 = x - 8$ {rational numbers}
   b. $4x + 3 = x - 8$ {integers}
8. a. $7x - 4 = 2x - 19$ {rational numbers}
   b. $7x - 4 = 2x - 19$ {integers}
9. a. $x^2 = 16$ {negative numbers}
   b. $x^2 = 16$ {real numbers}
10. a. $x^2 = 36$ {positive numbers}
    b. $x^2 = 36$ {real numbers}
11. a. $x^2 = 1/9$ {rational numbers}
    b. $x^2 = 1/9$ {integers}
12. a. $x^2 = 7$ {rational numbers}
    b. $x^2 = 7$ {irrational numbers}
13. $(x + 3)(3x - 2) = 0$ {real numbers}
14. $(2x - 5)(x + 1) = 0$ {real numbers}
15. $(2x - 5)(x + 1) = 0$ {integers}
16. $(x + 3)(3x - 2) = 0$ {integers}
17. $(x + 3)(3x - 2) = 0$ {positive numbers}
18. $(2x - 5)(x + 1) = 0$ {positive numbers}
19. $(2x + 3)(x - 5)(x + 1) = 0$ {real numbers}
20. $(4x - 3)(x + 2)(x - 6) = 0$ {real numbers}
21. $x(2x - 1)(x + 4) = 0$ {real numbers}
22. $x(5x - 3)(x - 6) = 0$ {real numbers}

For Problems 23 through 32, solve the equation assuming that the domain is $x \in$ {real numbers}.

23. $|x| = 7$
24. $|x| = 5$
25. $|x| = -6$
26. $|x| = -9$

27. $|x + 3| = 5$

28. $|x - 2| = 14$

29. $|4x - 1| = 11$

30. $|5x + 3| = 7$

31. $|9x - 17| = 1$

32. $|7x - 2| = 4$

33. Although they do not look much alike, there *is* a relationship between the equations $(x + 7)(3x - 5) = 0$ and $|3x + 8| = 13$.
    a. Find the solution set of each equation.
    b. What word best describes the relationship between the two equations?

34. Given the equation $(x - 3)(x - 7) = 0$:
    a. Write the solution set.
    b. Divide both members of the equation by $(x - 3)$, and write the solution set of the transformed equation.
    c. Is the transformed equation *equivalent* to the original one? Explain.
    d. Multiply both members of the original equation by $(x - 2)$. What number is a solution of the transformed equation which was *not* a solution of the original one?
    e. What name is given to the solution in part d?
    f. What name is given to the operation of multiplying both members of an equation by an expression which can equal 0?

35. ***Introduction to Inequalities*** — The statement $5 < 7$ is an *inequality*. It is *true* because 5 *is less* than 7.
    a. Add the positive number 3 to both members. Is the resulting inequality true?
    b. Add the negative number -8 to both members of the original inequality. Is the resulting inequality true?
    c. Use variables to write a statement of the *Addition Property of Order*, which states that you may add the same number to both members of an inequality *without* changing the order (from $<$ to $>$, for example).

36. ***More about Inequalites*** — Given the true inequality $5 < 7$:
    a. Multiply both members of the inequality by the positive number 3. Is the resulting inequality true?
    b. Multiply both members of the original inequality by the negative number -8. Is the resulting inequality true?
    c. Multiply both members of the original inequality by 0. Is the resulting inequality true?
    d. Write a statement of the *Multiplication Property of Order* which takes into account what you have observed in parts a, b, and c.
    e. In your own words, tell what happens to the order in an inequality when you multiply both members by a *negative* number. Tell also what *you* must do to make a true statement out of the resulting inequality.

1-6 INEQUALITIES

If the "=" sign in an equation is replaced by one of the order signs, $<$, $>$, $\leq$, or $\geq$, the resulting sentence is called an *inequality*. For example,

$$3x - 5 < 16.$$

The solution set of an inequality contains all values of the variable that make the sentence true. Since the solution set usually contains an *infinite* number of solutions, it is customary to draw a graph rather than write the set.

Objective:

Given an inequality, transform it to a simpler, equivalent inequality so that you can draw a graph of its solution set.

If you cannot see the solution set by inspection, you can transform the inequality by

1. adding the same number to both members, or
2. multiplying both members by the same number. (This is tricky! See Example 2.)

EXAMPLE 1

Graph the solution set of $3x - 5 < 16$.

Solution:
Starting with

$$3x - 5 < 16,$$

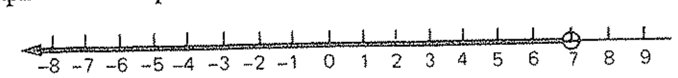
you would add 5 to each member, getting

$$3x < 21.$$

Then you would multiply each member by $\frac{1}{3}$, getting

$$x < 7.$$

The graph would be plotted on a number line, looking like this:



The open circle at 7 indicates that the endpoint of the ray is *not* included. A *closed* circle would be used for inequalities with $\leq$ or $\geq$, where the endpoint *is* included.

Suppose you were asked to graph the solution set of

$$-2x \geq 18.$$

To get rid of the "-2" on the left, you must multiply each member of the inequality by $-\frac{1}{2}$. But multiplying each member of an inequality by a *negative* number makes the order *reverse*. You can see why, by multiplying both members of an inequality like $5 < 7$ by a negative number such as -8. The result will be that -40 is *greater* than -56. So *you* must reverse the order sign whenever you multiply both members of an inequality by a negative number. This fact is summarized in the Multiplication Property of Order.

PROPERTY

Multiplication Property of Order

If $x < y$, then

$xz < yz$, if z is *positive*,
$xz > yz$, if z is *negative*,
$xz = yz$, if z is *zero*.

A similar property applies to the relationship $>$.

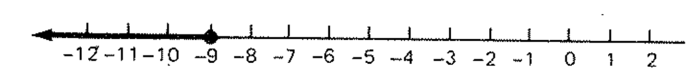
EXAMPLE 2

Graph the solution set of $-2x \geq 18$.

Solution:
The steps in solving the inequality would be:

$-2x \geq 18$	Write the given inequality.
$-\frac{1}{2}(-2x) \leq -\frac{1}{2}(18)$	Multiply by $-\dfrac{1}{2}$ and reverse the order sign.
$x \leq -9$	Associate and do the arithmetic.

The graph would be



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EXAMPLE 3

Graph the solution set of $3 \leq 2x + 5 < 11$.

Solution:
This inequality has *three* members. Starting with

$$3 \le 2x + 5 < 11,$$

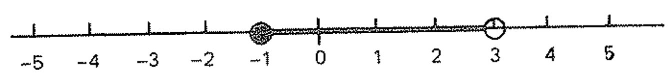
you would add -5 to *all three* members, getting

$$-2 \le 2x < 6.$$

Multiplying all three members by $\frac{1}{2}$ gives

$$-1 \le x < 3.$$

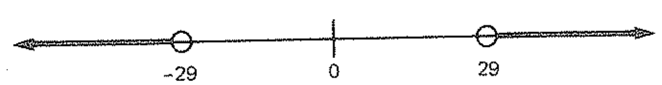
The graph is all numbers between -1 and 3, including the -1 but not including the 3.



EXAMPLE 4

Graph the solution set of $|x| > 29$.

Solution:
$|x|$ means, "the distance between the origin and x." So the inequality really says, "x is a number that is *more than* 29 units from the origin." Therefore, one part of the graph will start at 29 and go "upward," and the other part will start at -29 and go "downward." The graph is



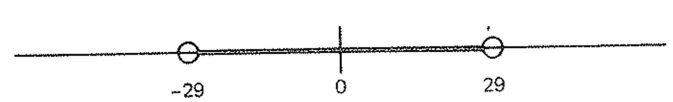
From the graph you should be able to tell that the inequality $|x| > 29$ is equivalent to the *combined* inequalities,

$$x > 29 \text{ or } x < -29$$

EXAMPLE 5

Graph the solution set of $|x| < 29$.

The solution set will contain all numbers that are *closer* to the origin than 29 units. So the graph will be all those points *between* -29 and 29. The graph is



From this graph, you should be able to tell that $|x| < 29$ is equivalent to

$$x < 29 \text{ and } x > -29,$$

which is equivalent to

$$-29 < x < 29.$$ ■

From Examples 4 and 5, you can figure out a way to transform inequalities to eliminate the absolute value sign.

CONCLUSION

> If c is a non-negative constant, then
> $|\text{expression}| > c$ is equivalent to:
>
> expression $> c$ *or* expression $< -c$.
>
> $|\text{expression}| < c$ is equivalent to:
>
> expression $< c$ *and* expression $> -c$.
> (or to $-c <$ expression $< c$).

If you ever forget these transformations, you can think them up easily by recalling that the absolute value of a number is its distance from the origin.

EXAMPLE 6

Graph the solution set of $|3x - 5| \leq 13$ if the domain of x is

a. {real numbers},
b. {positive numbers},
c. {integers}.

Starting with

$$|3x - 5| \leq 13,$$

you use the appropriate transformation, above, to write

$$-13 \leq 3x - 5 \leq 13.$$

Adding 5 to all three members,

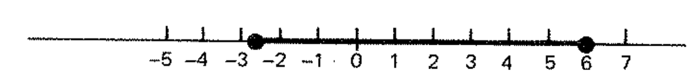
$$-8 \leq 3x \leq 18.$$

Dividing all three members by 3 gives

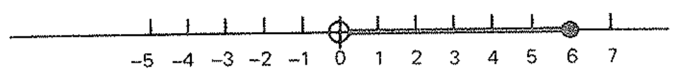
$$-\tfrac{8}{3} \leq x \leq 6.$$

The graphs are as follows.

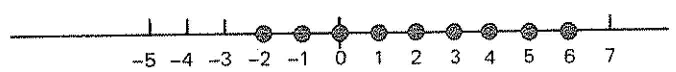
a. {real numbers}



b. {positive numbers}



c. {integers}



The following exercise is designed to give you practice in graphing the solution sets of inequalities.

EXERCISE 1-6

Do These Quickly

The following problems are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. Solve: $4x + 8 = 28$

Q2. Simplify: $4x + 8 - 28$

Q3. Simplify: $4(x + 8) - 28$

Q4. Find 10% of 1234.

Q5. Multiply and simplify: $(\frac{5}{3})(\frac{3}{5})$

Q6. What axiom is illustrated? $3(x + 4) \cdot 1 = 3(x + 4)$

Q7. Is $\frac{3}{5}$ an integer?

Q8. Name by degree and number of terms: $x^3 - 17^4$

Q9. Solve: $|x - 2| = 7$

Q10. Evaluate if x is 14: $|13 - x|$

For Problems 1 through 14, graph the *solution* set of the inequality, observing the given domain.

1. $3x - 5 < 4$ {real numbers}
2. $4x - 7 > 1$ {real numbers}
3. $2 - 6x \leq 8$ {real numbers}

4. $5 - 3x \geq -4$	{real numbers}
5. $2x + 3 > 10$	{integers}
6. $3x - 5 < 2$	{integers}
7. $-6 < x - 5 \leq 2$	{real numbers}
8. $4 \leq x - 1 < 7$	{real numbers}
9. $x + 3 \geq 2$ or $x + 3 < -7$	{real numbers}
10. $x - 4 > -1$ or $x - 4 \leq -6$	{real numbers}
11. $x + 2 < 5$ or $x + 2 \geq 7$	{positive numbers}
12. $3 \leq x + 4 < 5$	{negative numbers}
13. $3 - 2x > 5$	{positive numbers}
14. $4 - 3x < -2$	{negative numbers}

For Problems 15 through 32, graph the solution set assuming that the domain is:

a. $x \in$ {real numbers}
b. $x \in$ {integers}
c. $x \in$ {positive numbers}

15. $\lvert x \rvert < 5$	16. $\lvert x \rvert \leq 7$
17. $\lvert x \rvert \geq 2$	18. $\lvert x \rvert > 3$
19. $\lvert x + 2 \rvert \leq 3$	20. $\lvert x - 1 \rvert < 4$
21. $\lvert 2x + 5 \rvert > 9$	22. $\lvert 3x + 7 \rvert \geq 7$
23. $\lvert 5x - 6 \rvert \leq 16$	24. $\lvert 4x - 3 \rvert < 7$
25. $\lvert 2x - 7 \rvert \geq 21$	26. $\lvert 5x - 3 \rvert > 22$
27. $\lvert 4x + 2 \rvert < 9$	28. $\lvert 2x + 4 \rvert \leq 9$
29. $\lvert x - 1 \rvert < -3$	30. $\lvert x + 3 \rvert > -6$
31. $3 \leq \lvert x - 2 \rvert < 5$	32. $4 < \lvert x - 3 \rvert \leq 7$

1-7 PROPERTIES PROVABLE FROM THE AXIOMS

The Field Axioms you studied in Section 1-2 express properties of addition and multiplication which should be obvious to you. However, there are some true properties which are *not* obvious, and some things which seem obvious, but are *not true*. Therefore, mathematicians seek ways of proving new properties from a small handful of axioms.

The properties you will be proving in this section form the basis for the mathematics you will study for the rest of the course. Some you already know. The others you can either prove *now*, or you can go on to Chapter 2 and return to these proofs as you need the properties.

Objectives:

1. Given the steps in the proof of a new property, name a property justifying each step.
2. Given the name or statement of a new property and some clues about how to prove it, prove the property giving reasons for each step.

You must first recall the axioms which apply to the relationships =, <, and > . These are as follows.

AXIOM

REFLEXIVE PROPERTY
If x is a real number, then $x = x$.

Equality is said to be a *reflexive* relationship. The name is picked because a number can "look into the '=' sign" and see its own "reflection" on the other side. It is this axiom which expresses the fact that a variable stands for the *same* number, wherever it appears in an expression. Note that order is *not* reflexive since statements like $5 < 5$ or $5 > 5$ are *false*.

AXIOM

SYMMETRY
If $x = y$, then $y = x$.

Equality is said to be a *symmetric* relationship. The "=" sign looks the same when viewed from either direction. So it does not matter on which side of the "=" sign a number appears. The order relationships are *not* symmetrical. For example, the statement $4 < 5$ *cannot* have the numbers reversed. The statement $5 < 4$ is *false*.

AXIOM

> **TRANSITIVITY**
> For Equality: If $x = y$ and $y = z$, then $x = z$.
> For Order: If $x < y$ and $y < z$, then $x < z$.
> If $x > y$ and $y > z$, then $x > z$.

The prefix "trans-" means, "across," "beyond," or "through." For example, a rapid *transit* system carries you *through* a city. The name is used here because the equality or order carries through from the first number to the last one. The property, extended, can be used when you are simplifying an expression to connect the original form to the final form. For example:

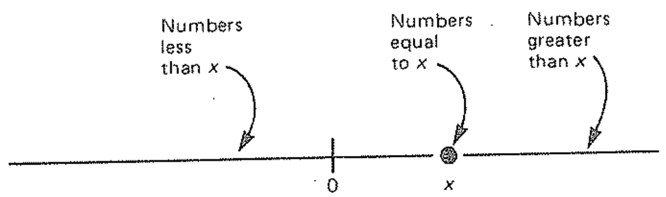
$$\begin{aligned} &(x + 3)(x + 4) \\ &= x(x + 4) + 3(x + 4) \\ &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \\ \therefore\ &(x + 3)(x + 4) = x^2 + 7x + 12 \leftarrow \text{Transitivity used } \textit{here}. \end{aligned}$$

Note that the "=" signs on the second and third lines connect the expression to the one *just above* it, not to the original expression. This is why the transitive step is needed at the end. See Appendix B for a proof of this "extended" transitive property.

AXIOM

> **TRICHOTOMY**
> If x and y are two real numbers, then *exactly one* of the following must be true:
>
> $y < x$
> $y > x$
> $y = x$.

This axiom tells you how any two given numbers must *compare* to each other. The words, "exactly one" mean two things. *One* of the statements *must* be true, and *only one* of the statements can be true. The word "trichotomy," meaning "cut in three," is used because placing a number x on the number line *cuts* it into *three* pieces.



Any other number, y, must be on one of these three pieces.

These axioms help you with the mechanics of going from one step to the next in a proof. You are now armed with the tools you need to do some proving.

EXAMPLE 1

Substitution into Sums and Products (*not* an axiom)

Prove that if $z = x$, then

$x + y$ and $z + y$ stand for *equal* numbers,
xy and zy stand for *equal* numbers.

(In plain English, this property says that equals can be substituted for equals in sums or products.)

Proof:

The Closure Axiom states that a sum such as $x + y$ or a product such as xy stands for a *unique* real number. For this reason, it does not matter what other symbol you use for the number x. Therefore, $x + y = z + y$, and $xy = zy$, Q.E.D.

This is called a *paragraph proof*. You have written a paragraph explaining why a certain property is true. The letters "Q.E.D." at the end stand for the Latin words, "quod erat demonstrandum," meaning, "which was to be proved."

EXAMPLE 2

Addition Property of Equality (not an axiom)

Prove that if $x = y$, then $x + z = y + z$.

(In plain English, prove that you can add the same number to each member of an equation.)

Proof:

a. $x + z = x + z$	a. Reflexive Axiom.
b. $x = y$	b. Hypothesis (given).
c. $\therefore x + z = y + z$, Q.E.D.	c. Substitution into a sum.

Proofs such as this one are not easy to think up from "scratch." However, there is usually a "key step" that forms the heart of the proof. In this case, the thought process is, "I want an equation that has $x + z$ on the left side, so I'll *write* an equation like that and *transform* it to the one I want." The substitution property gives you a way to do the transformation.

The "if" part of a property is called the "hypothesis." The word comes from "hypo-" (as in hypodermic), meaning "under," and "thesis," (as in thesis sentence), meaning "main idea." The property you get by *reversing* the hypothesis and the conclusion is called the *converse* of the property. The converse of a true statement may or may not be true. For example,

"If you play varsity football, then you are male,"

is (probably!) a true statement. But its converse,

"If you are a male, then you play varsity football,"

is *not* true. In the next example is a proof of the converse of the Addition Property of Equality.

EXAMPLE 3

Converse of the Addition Property of Equality

Prove that if $x + z = y + z$, then $x = y$.

Proof:

a. $x + z = y + z$	a. Given ("hypothesis")
b. $(x + z) + (-z) = (y + z) + (-z)$	b. Addition property of equality.
c. $x + (z + (-z)) = y + (z + (-z))$	c. Associativity for addition.
d. $x + 0 = y + 0$	d. Additive inverses.
e. $x = y$, Q.E.D.	e. Additive identity.

In this proof it was helpful to start with the hypothesis and work toward the conclusion. Note that you can *never* start with the conclusion, nor can you use the conclusion as a reason in the proof. Doing so is called "circular reasoning," for which your instructor will probably give you circular grades!