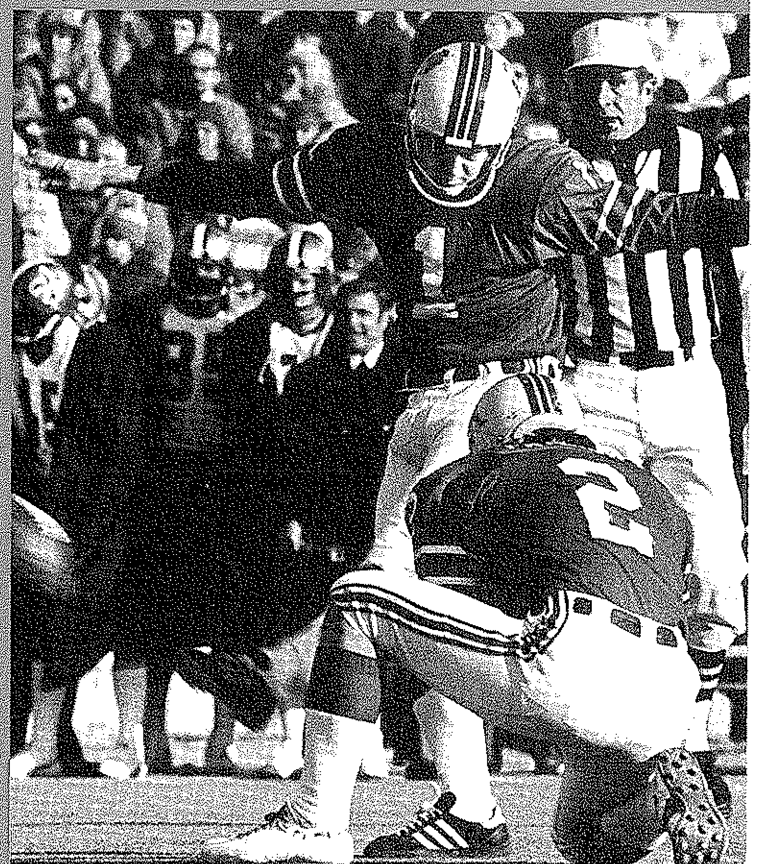
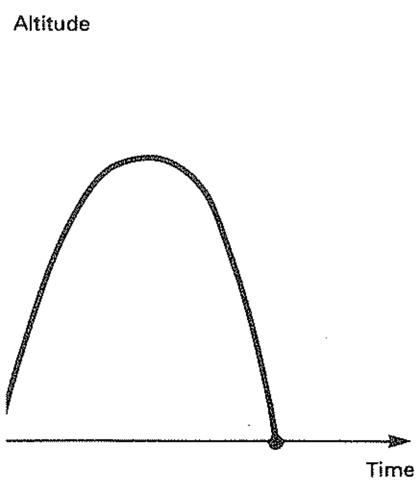
2

Functions and Relations



'n Chapter 1 you refreshed your nemory about properties of num- ɔers, and how the properties are ɩsed to evaluate expressions and ;olve equations. Now you will ;oncentrate on equations that ɩave ***two*** *variables. These equa- ions tell how one variable is* ***re- ated*** *to the other. In Exercise ?-3 you will draw graphs show- ng the relationship betwen vari- ɩbles such as the altitude of a ɔotball and the time since it was :icked, or the temperature of the vater and the time since the 'hot" faucet was turned on.*



c. $(x - y) + (y - z)$ is positive.
d. $x + (-y + y) - z$ is positive.
e. $x + 0 - z$ is positive.
f. $x - z$ is positive.
g. $x > z$, Q.E.D.

T13. Prove that if you multiply both members of an inequality by a *positive* number, the order does *not* change. That is, prove that if $x > y$ and z is positive, then $xz > yz$.

T14. Supply reasons that the order *reverses* in the steps in the following proof when you multiply both members of an inequality by a *negative* number.

Prove that if $x > y$ and z is negative, then $xz < yz$.

Proof:

a. z is negative.
b. $\therefore (-1)(z)$ is positive.
c. $\therefore -z$ is positive.
d. $x > y$
e. $\therefore x - y$ is positive.
f. $\therefore (x - y)(-z)$ is positive.
g. $\therefore -xz + yz$ is positive.
h. $\therefore yz + (-xz)$ is positive.
i. $\therefore yz - xz$ is positive.
j. $\therefore yz > xz$.
k. $\therefore xz < yz$, Q.E.D.

T15. Use the result of Problem Concepts Test 14 as a lemma to prove that if $x < y$ and z is negative, then $xz > yz$.

2-1 GRAPHS OF EQUATIONS WITH TWO VARIABLES

You have learned how to plot number-line graphs of equations with *one* variable. In this section you will plot a graph of an equation with *two* variables.

Objective:

Use what you recall from previous mathematics courses to draw the graph of an equation with two variables.

In order to satisfy the equation

$$2x - 3y = 15,$$

it is necessary to specify values for both x and y. For instance, if $x = 9$, then

$$2 \cdot 9 - 3y = 15 \quad \text{Substituting 9 for } x.$$

$$-3y = -3 \quad \text{Subtracting 18.}$$

$$y = 1. \quad \text{Dividing by } -3.$$

So $x = 9$ and $y = 1$ is a solution of $2x - 3y = 15$.

It is customary to write the values of the two variables as *ordered pairs,* such as (9, 1), where the first number stands for a value of x and the second for a value of y.

The solution set of an equation with two variables contains all the ordered pairs that make the equation true. Since there are *two* numbers in each ordered pair, the graph will consist of points in a *two*-dimensional plane, as in Figure 2-1, rather than a one-dimensional number line. The first coordinate ("abscissa") in the ordered pair is plotted horizontally and the second

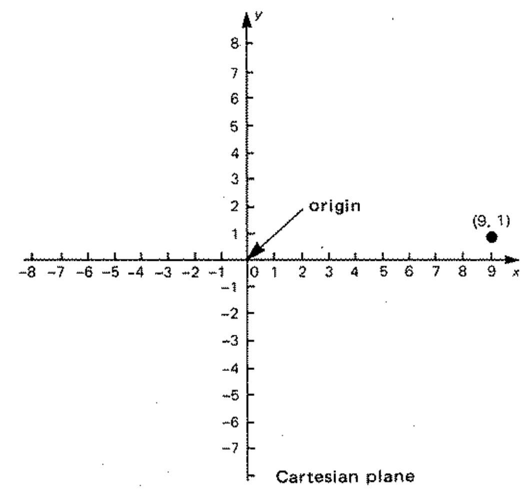


Figure 2-1

coordinate ("ordinate") is plotted vertically. You may recall from previous mathematics courses that this plane is called a Cartesian coordinate system, after the French mathematician Rene Descartes who lived from 1596 to 1650.

In the following exercise you will plot more solutions of the equation $2x - 3y = 15$, and try to see what pattern the points follow.

EXERCISE 2-1

1. Show that the ordered pair (9, 1) satisfies the equation $2x - 3y = 15$. Do this by substituting 9 for x and 1 for y, and showing that you get a *true* statement.
2. Show that the ordered pair (1, 9) does *not* satisfy the equation $2x - 3y = 15$.
3. Substitute 6 for x in $2x - 3y = 15$, and solve for y. Then draw a Cartesian coordinate system as in Figure 2-1, and plot this point on it.
4. Repeat Problem 3 for $x = 3$, 0, and -3. Use the same Cartesian coordinate system.
5. Connect the points you have drawn on the Cartesian coordinate system of Problem 3. If they do not all lie on the same straight line, go back and check your work!

2-2 GRAPHS OF FUNCTIONS

In Exercise 2-1 you plotted the graph of the solution set of $2x - 3y = 15$. The graph should look like Figure 2-2a.

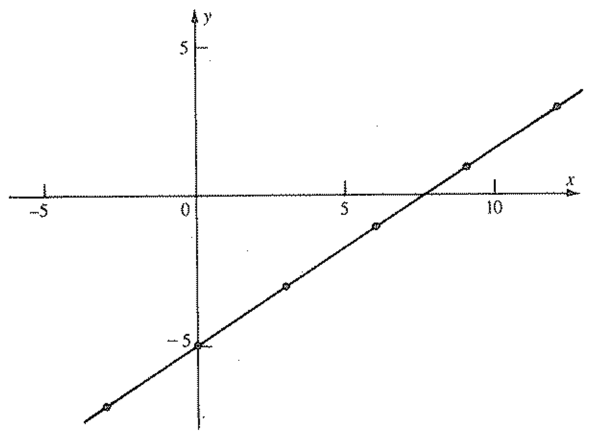


Figure 2-2a

The graph can be more easily plotted if you first transform the equation so that y is by itself on one side.

$2x - 3y = 15$	Write the given equation.
$-3y = -2x + 15$	Subtract $2x$ from each member.
$y = \frac{2}{3}x - 5$	Divide each member by -3.

All you need to do to find many ordered pairs is pick values of x (preferably multiples of 3 in this case), substitute them, and calculate y. The dots in Figure 2-2a show some of these ordered pairs.

Whenever a unique value of y can be found for each value of x, y is said to be a *function* of x. The formal definition of function is in Section 2-4. The variable that appears by itself on one side of the equation is called the *dependent variable* because the value you get for it depends on what you picked for the other variable. The other variable is called the *independent variable*. The dependent variable is usually plotted on the vertical axis.

Objective:

Given the equation of a function, plot the graph.

The graph in Figure 2-2a is a straight line. Many functions have graphs that are curved. In order to plot such graphs you should evaluate y for enough values of x to find a pattern. Then connect the points with a smooth curve.

EXAMPLE 1

Plot the graph of $y = 0.2x^2$.

Solution:
Make a table of values, then plot these as shown in Figure 2-2b.

x	y
−5	5
−4	3.2
−3	1.8
−2	0.8
−1	0.2
0	0
1	0.2
2	0.8
3	1.8
4	3.2
5	5

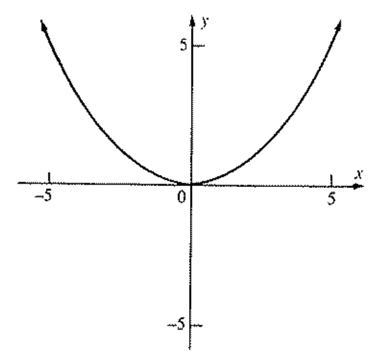


Figure 2-2b

Sometimes not all values of x are permitted. For instance, in the function $y = \frac{1}{(x-2)}$, x cannot be 2 or you would wind up dividing by zero. At other times you are simply told that the function applies only for certain values of x. The values of x that you are allowed to substitute is called the domain of the function.

DEFINITION

DOMAIN
The **domain** of a function is the set of values of the *independent variable*.

As was pointed out in Section 1-3, the word "domain" comes from the Latin word "domus," meaning "house." So the domain of a function is where the independent variable "lives."

The set of values you would get for y by substituting all permissible values of x is called the range of the function.

DEFINITION

RANGE
The *range* of a function is the set of values of the *dependent* variable corresponding to all values of the independent variable in the domain.

The graph in Figure 2-2c is intended to help you keep in mind the definitions of domain and range.

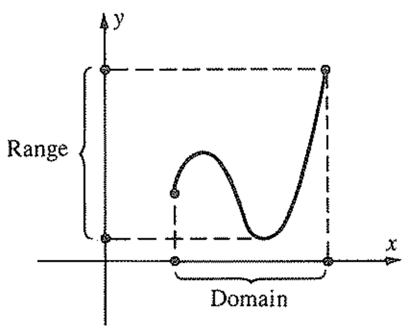


Figure 2-2c

EXAMPLE 2

Plot the graph of $y = \frac{10}{x}$ if the domain is {integers between 1 and 6, inclusive}. Tell the range.

Solution:

x	y
1	10
2	5
3	$3\frac{1}{3}$
4	$2\frac{1}{2}$
5	2
6	$1\frac{2}{3}$

Range = $\{10, 5, 3\frac{1}{3}, 2\frac{1}{2}, 2, 1\frac{2}{3}\}$

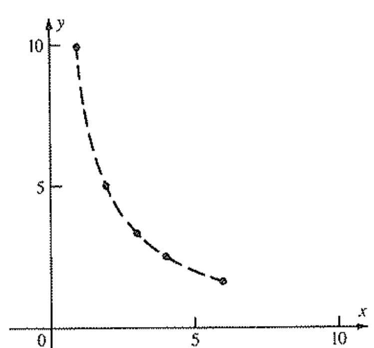


Figure 2-2d

The graph is shown in Figure 2-2d. Note that the graph is a set of discrete points. The dotted line connecting the points just shows the pattern they follow, and is not part of the graph itself.

In the following exercise you will plot graphs of functions.

EXERCISE 2-2

Do These Quickly

The following are 10 miscellaneous problems. They are intended to refresh your skills from the first chapter and from previous courses. You should be able to do all 10 in less than 5 minutes.

Q1. Tell what axiom was used: $4(3 + x) = 4(x + 3)$

Q2. Solve: $8x + 3 = 39$

Q3. Simplify: $5x - 2 + 6x$

Q4. Distribute: $7x(x^3 - 2)$

Q5. Find 70% of 42.

Q6. Write $\frac{3}{8}$ as a decimal.

Q7. Solve: $|x - 2| = 8$

Q8. Isolate x: $-5x > 34$

Q9. Write a negative rational number that is not an integer.

Q10. Tell the degree: $5^3x^9 - y^4z^6$

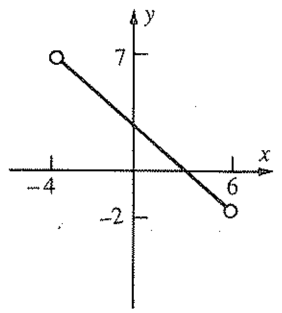
For Problems 1 through 12, plot the graph of the function and tell the range.

1. $y = 0.3x^2$, domain = {real numbers}
2. $y = -0.5x^2$, domain = {real numbers}
3. $y = x + 3$, domain = {non-negative integers}
4. $y = x - 5$, domain = {positive integers}
5. $y = \frac{-12}{x}$, domain = {positive real numbers}

6. $y = \frac{5}{x}$, domain $= \{x: 0.4 \le x \le 10\}$
7. $y = \frac{2}{3}x + 4$, domain $= \{x: -3 < x < 9\}$
8. $y = -0.4x + 5$, domain $= \{x: -2 \le x \le 6\}$
9. $y = |x - 3|$, domain $= \{x: 0 \le x \le 7\}$
10. $y = |x + 2|$, domain = {real numbers}
11. $y = x^2 - 5x + 7$, domain $= \{0, 1, 2, 3, 4, 5, 6\}$
12. $y = -x^2 + 5.4x + 1$, domain = {positive numbers}

For Problems 13 through 16, tell the domain and range of the function.

13.



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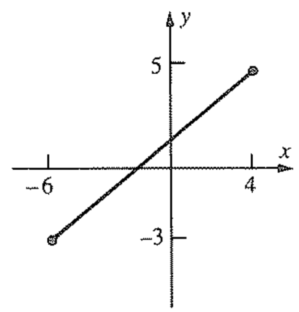
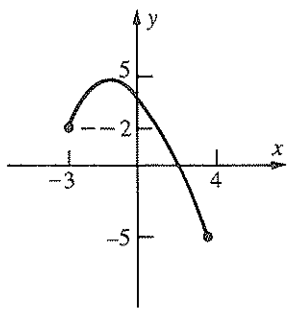
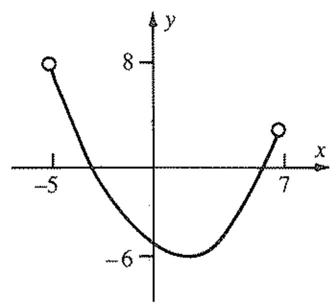


15.



16.



For Problems 17 through 20, sketch a graph of a function with the given domain and range.

17. domain: $\{x: 3 \le x \le 7\}$, range: $\{y: 1 \le y \le 10\}$
18. domain: $\{x: 1 \le x \le 4\}$, range: $\{y: -3 \le y \le 5\}$
19. domain: $\{x: 2 < x < 3\}$, range: $\{y: 5 < y < 7\}$
20. domain: $\{x: 0 < x < 5\}$, range: $\{y: 2 < y \le 7\}$ (Be clever!)

2-3 FUNCTIONS IN THE REAL WORLD

In the last section you plotted graphs of functions that had equations such as

$$y = \frac{10}{x}.$$

In situations from the real world there are often two variable quantities that are related in such a way that the value of one variable depends on the value of the other. For example:

1. The position of a speedometer needle depends on how fast the car is going.
2. The distance you travel depends on how long you have been traveling (and on how fast you are going, also!).
3. The weight of a person depends on his or her height (and on other variables).
4. How badly your thumb hurts depends on how hard you hit it with a hammer.

In cases like this you may say, for example, that the distance traveled is a *function* of time. If you know something about the relationship between distance and time, you may be able to write an equation relating the variables. Even if you don't know enough to write an equation, you can still draw a reasonable graph representing the relationship. In this section you will sketch this kind of graph.

Objective:

Given a situation from the real world in which the value of one variable depends on the value of the other, sketch a reasonable graph showing this relationship.

EXAMPLE 1

The time it takes you to get home from the football game and the speed you drive are related to each other. Sketch a reasonable graph showing this relationship.

Solution:

Since you are not told which variable depends on the other, your first job is to make this decision. You should ask yourself which of the following sounds more reasonable:

"How long it takes depends on how fast I go." "How fast I go depends on how long it takes."

Most people feel that the first sentence is more reasonable, and pick *time* as the dependent variable. So speed is the independent variable. Since you have already agreed to plot the dependent variable on the vertical axis, you draw and label axes as in Figure 2-3a.

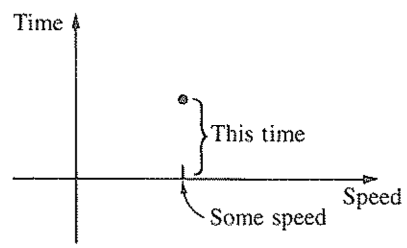


Figure 2-3a

To figure out what the graph looks like, pick some moderate speed and plot a point at a moderate length of time, as in Figure 2-3a. Then think about what happens to the time as your speed varies. When speed is *lower*, time is *longer*. When speed is *higher*, time is *shorter*. Put a point to the left and above the first one, and another to the right and below the first one, as in Figure 2-3b.

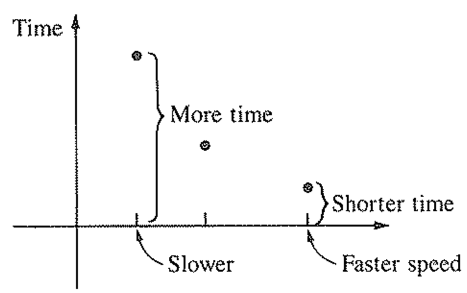


Figure 2-3b

When you have enough points to tell what the graph looks like, connect them with a line or curve. Figure 2-3c shows the completed graph. Since it always takes you *some* amount of time no matter how fast you drive, the graph never touches the horizontal axis. Similarly, since you would *never* get home if speed were zero, the graph never touches the vertical axis.

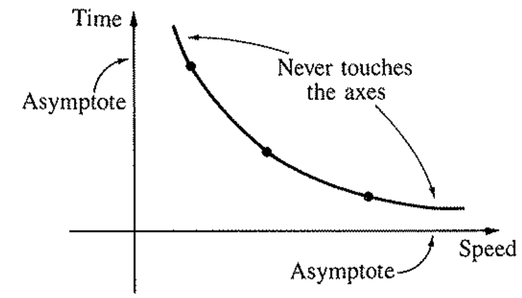


Figure 2-3c

A line the graph approaches as it does the horizontal or vertical axis in Figure 2-3 is called an *asymptote*. The word comes from Greek, and means, "not coming together."

DEFINITION

> **ASYMPTOTE**
> An **asymptote** is a line which a graph gets arbitrarily close to, but never touches, as the independent or dependent variable gets very large (in the positive or the negative direction).

In Example 1, the speed should always be positive. Negative speeds have no meaning in this case since going "backwards" would not take you home. So the domain of this function is

$$\text{Domain} = \{\text{speed} > 0\}.$$

Only positive values of time make sense in this example. You cannot get home *before* you start or at the *instant* you start. So the range of the functioin, corresponding to the domain, is

$$\text{Range} = \{\text{time} > 0\}.$$

EXAMPLE 2

You take a roast beef from the refrigerator and put it into a hot oven. The temperature of the beef depends on how long it has been in the oven. Sketch a reasonable graph.

Solution:
Figure 2-3d shows a reasonable graph. When time < 0, the beef is still in the refrigerator, so its temperature is the same as that of the refrigerator. For time > 0, the beef warms up rapidly at first, then more slowly, and finally approaches the oven temperature very gradually. It is debatable whether the beef ever actually *reaches* oven temperature, or just gets so close that nobody can tell the difference. Thus, the dotted line at oven

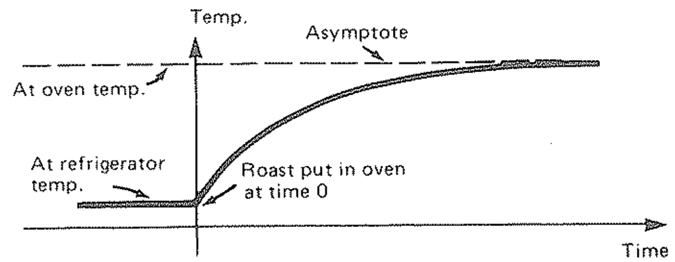


Figure 2-3d

temperature is an *asymptote*. The domain in this case includes both positive and negative values of time. The range is the set of temperatures between refrigerator temperature and oven temperature. The temperatures *could* be *negative* if the beef had been in the freezer. You might be able to think of ways to make the graph even more reasonable in that case!

In the exercise which follows, you will obtain practice sketching reasonable graphs of real-world situations in accordance with the objective of this section. Many of these real-world situations will appear in later chapters when you study about relations which have graphs like these.

EXERCISE 2-3

Do These Quickly

The following problems are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. On which axis is the independent variable plotted?

Q2. Draw an isosceles triangle.

Q3. Find 3% of 800.

Q4. Multiply: $(2x + 7)(3x - 4)$

Q5. If j is a function of p, which variable *depends*?

Q6. Multiply: $(0.3)(0.2)$

Q7. Add and simplify: $\frac{2}{3} + \frac{3}{4}$

Q8. Associate the 7 and the y: $4 + 7 + y$

Q9. Solve: $|x - 4| = 13$

Q10. Draw a number-line graph: $-x < 5$

For Problems 1 through 44, sketch a reasonable graph.

Sketch a reasonable graph.

1. The distance you have gone depends on how long you have been going (at a constant speed).
2. The number of used aluminum cans you collect and the number of dollars refunded to you are related.
3. The distance required to stop your car depends on how fast you are going when you apply the brakes.
4. The mass of a person of average build depends on his or her height.
5. Your car is standing on a long, level highway. You start the motor and floorboard the gas pedal. The speed you are going depends on the number of seconds that have passed since you stepped on the gas pedal.
6. The altitude of a punted football depends on the number of seconds since it was kicked.
7. The maximum speed your car will go depends on how steep a hill you are going up or down.
8. Dan Druff's age and the number of hairs he has growing on his head are related.
9. The distance you are from the band and how loud it sounds to you are related.
10. You fill up your car's gas tank and start driving. The amount of gas you have left in the tank depends on how far you have driven.
11. Your age and your height are related to one another.
12. You pull the plug out of the bathtub. The amount of water remaining in the tub and the number of seconds since you pulled the plug are related to each other.
13. The price you pay for a carton of milk depends on how much milk the carton holds.
14. Calvin Butterball desires to lose some weight, so he reduces his food intake from 8000 calories per day to 2000 calories per day. His weight depends on the number of days that have elapsed since he reduced his food intake.
15. The price you pay for a pizza depends on the diameter of the pizza.
16. The distance you are from the reading lamp and the amount of light it shines on your book are related.
17. You climb to the top of the 190-meter tall Tower of the Americas and drop your algebra book off. The distance the book is above the

ground depends on the number of seconds that have passed since you dropped it.

18. As you blow up a balloon, its diameter and the number of breaths you have blown into it are related.
19. The temperature of your cup of coffee is related to how long it has been cooling.
20. You turn on the hot water faucet. As the water runs, its temperature depends on the number of seconds it has been since you turned on the faucet.
21. The time of sunrise depends on the day of the year.
22. As you breathe, the volume of air in your lungs depends upon time.
23. You start running, and go as fast as you can for a long period of time. The number of minutes you have been running and the speed you are going are related to each other.
24. As you play with a yo-yo, the number of seconds that have passed and the yo-yo's distance from the floor are related.
25. You run the mile once each day. The length of time it takes you to run it depends on the number of times you have run it in practice.
26. When you dive off the 3-meter diving board, time, and your position in relation to the water's surface, are related to each other.
27. The rate at which you are breathing depends on how long it has been since you finished running a race.
28. Taryn Feathers catches the bus to work each morning. Busses depart every 10 minutes. The time she gets to work depends on the time she leaves home.
29. Milt Famey pitches his famous fast ball to Stan Dupp, who hits it for a home run. The number of seconds that have elapsed since Milt released the ball and its distance from the ground are related.
30. The amount of postage you must put on a first-class letter depends on the weight of the letter.
31. The amount of water you have put on your lawn depends on how long the sprinkler has been running.
32. You plant an acorn. The height of the resulting oak tree depends on the number of years that have elapsed since the planting.
33. A leading soft drink company comes out with a new product, Ms. Phizz. They figure that there is a relationship between how much of the stuff they sell and how many dollars they spend on advertising.
34. Your car stalls, so you get out and push. The speed at which the car goes depends on how hard you push.

35. The number of letters in the corner mailbox depends upon the time of day.
36. The diameter of a plate and the amount of food you can put on it are related to each other.
37. The number of cents you pay for a long distance telephone call depends on how long you talk.
38. The grade you could make on a particular test depends upon how long you study for it.
39. You go from the sunlight into a dark room. The diameter of your pupils and the length of time you have been in the room are related.
40. The grade you could make on a particular test depends on how much time elapses between the time you study for it and the time you take the test.
41. You pour some cold water from the refrigerator into a glass, but forget to drink it. As the water sits there, its temperature depends on the number of minutes that have passed since you poured it.
42. Your efficiency at studying algebra depends on how late at night it is.
43. You pour some popcorn into a popper and turn it on. The number of pops per second depends on how long the popper has been turned on.
44. How well you can concentrate on algebra homework and how late at night it is are related.

2-4 GRAPHS OF FUNCTIONS AND RELATIONS

So far the functions you have graphed have had a common feature. For any value of x you picked, there is only *one* value of y. Sometimes two variables are related by an equation that produces more than one value of y for a given value of x. For instance, if

$$y^2 = x,$$

then substituting a value such as 9 for x leads to

$$y^2 = 9$$

$$y = 3 \text{ or } -3.$$

Substituting a negative value for x produces no real value of y at all!

$$y^2 = -25$$

$$y = \text{no real number.}$$

Substituting other convenient values of x produces the following table.

x	y
0	0
1	1 or −1
4	2 or −2
9	3 or −3
−1	no real value
−4	no real value

The graph is shown in Figure 2-4a.

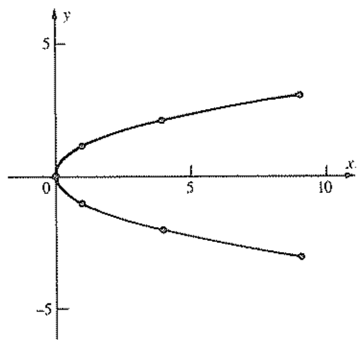


Figure 2-4a

If substituting a value of x ever produces more than one value for y, then y is not said to be a function of x. However, x and y are still related. Any set of ordered pairs relating two variables is called a *relation*. The name "function" is used only if each value of x has a unique value of y. This fact leads to the following formal definitions of relation and function.

DEFINITIONS

RELATION
A *relation* is a set of ordered pairs.

FUNCTION
A *function* is a relation for which there is *exactly one* value of the dependent variable for each value of the independent variable.

Objective:

Given the equation of a relation, draw its graph and tell whether or not the relation is a function.

EXAMPLE 1

Graph $|y| = x$. Tell whether or not the relation is a function.

Solution:
Substituting a positive value for x produces two values for y. For instance, if x is 4, then $|y| = 4$, and y is 4 or -4. Substituting a negative value for x produces no value for y. The equation $|y| = -3$ has no solutions. The graph is shown in Figure 2-4b.

x	y
-1	no value
0	0
1	1 or -1
2	2 or -2
3	3 or -3
4	4 or -4

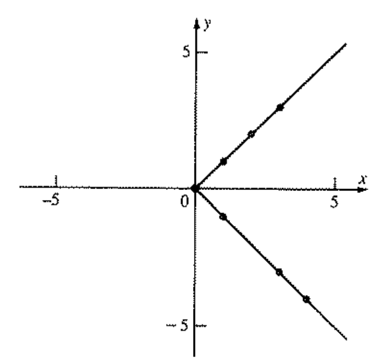


Figure 2-4b

Not a function.

If you already know what the graph of a relation looks like, you can tell quickly whether or not it is a function by using the *vertical line test*. Pick a value of x and draw a vertical line. If the graph ever crosses a vertical line more than once, then the relation is not a function. The result is shown in Figure 2-4c.

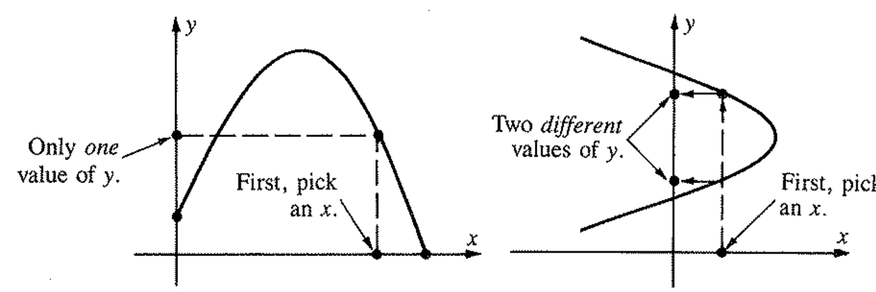


Figure 2-4c

The words "domain" and "range" are used for all relations, not just for those that are functions.

In the following exercise you will graph relations some of which are functions, and others of which are not.

EXERCISE 2-4

Do These Quickly

The following are 10 miscellaneous problems. They are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. Tell what axioms are used: $4(3 + x) = 4x + 12$

Q2. Solve: $2x - 6x = 14$

Q3. Simplify: $2x - 6x - 14$

Q4. Multiply: $(2x + 5)(3x - 1)$

Q5. 50 is 40% of what number?

Q6. Write $\frac{5}{8}$ as a decimal.

Q7. Draw a parallelogram.

Q8. Isolate x: $5x > -34$

Q9. Write an expression for 3 less than twice x.

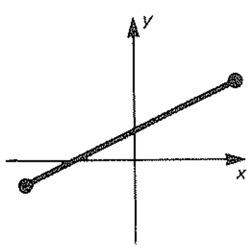
Q10. Find $|13|$.

For Problems 1 through 10, plot the graph and tell whether or not the relation is a function. Assume that the domain is the set of all values of x for which there are real-number values of y.

1. $y^2 = 9x$
2. $9y = x^2$
3. $y = |x - 3|$
4. $|y| = x + 2$
5. $2x + 3y = 12$
6. $5x - 2y = 10$
7. $|y| = 0.5x$
8. $y^2 = 4x$
9. $|y| = |x|$
10. $2y = x + |x|$

For Problems 11 through 26, tell whether or not the relation graphed is a function.

11.



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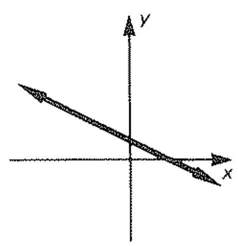

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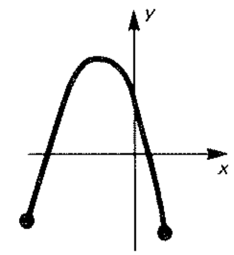

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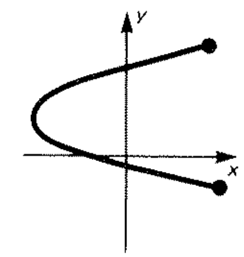

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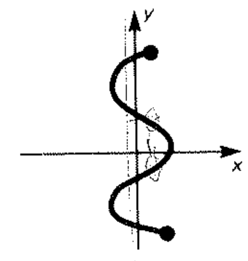

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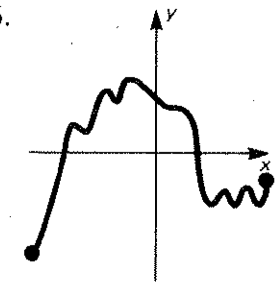

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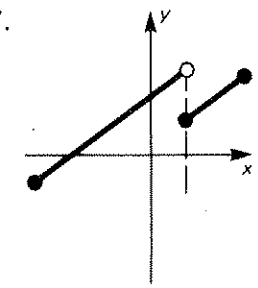

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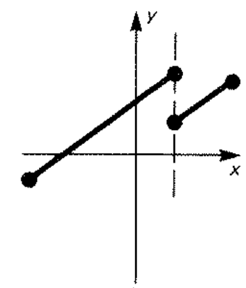

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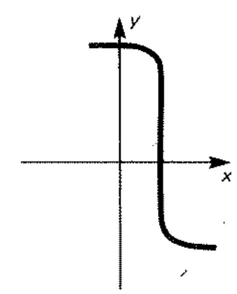

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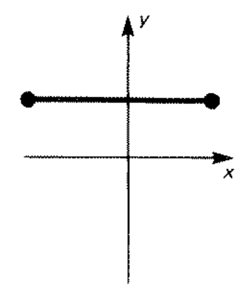

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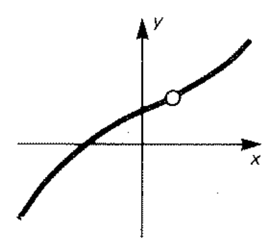

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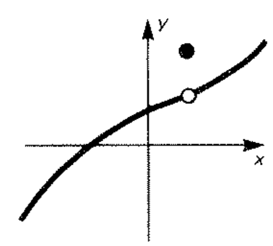

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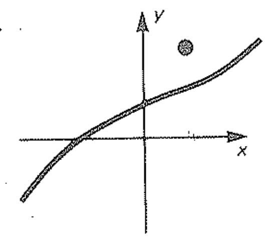

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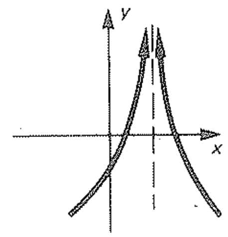

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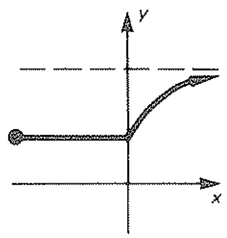

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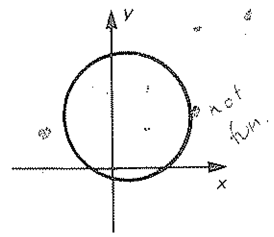


2-5 CHAPTER REVIEW AND TEST

In this chapter you have been introduced to the concept of a mathematical *function*. A function is a special kind of *relation*, or set of ordered pairs. A relation can be specified by an equation that tells how the two variables are related. The graph of such an equation can be plotted by calculating enough points to discover a pattern. Once you have plotted the graph, you can tell whether or not the relation is a function by seeing whether there are any values of x that have more than one value of y. Functions are important because they can be used to describe the relationship between two variable quantities in the real world.

The objectives of this chapter may be summarized as follows:

1. *Plot the graph of a given equation.*
2. *Tell whether or not a given graph is a function graph.*
3. *Draw reasonable graphs of real-world situations.*

Following are two sets of problems. The Review Problems are similar to those you have worked in this chapter. They are numbered according to the three objectives above. The Concepts Test requires you to put together two or more of these techniques (or to use concepts learned before), to work problems that are somewhat different. One of the most important things you should learn during your education is how to apply your knowledge to new situations!

REVIEW PROBLEMS

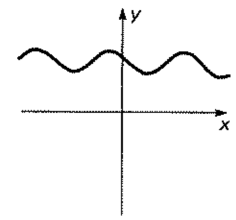
The following problems are numbered according to the three objectives listed above.

R1. Plot the graph of the given equation in the indicated domain. Tell the corresponding range and whether or not the relation is a function.

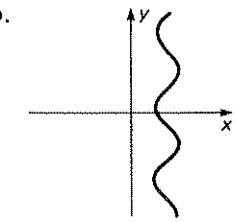
a. $x + y = 2, \quad -1 \le x \le 3$
b. $x^2 + y = 2, \quad x \in \{-2, -1, 0, 1, 2\}$
c. $x + y^2 = 2, \quad x \in \{-2, 1, 2\}$
d. $x + |y| = 2, \quad -2 \le x \le 2$

R2. Tell whether or not the relation graphed is a function.

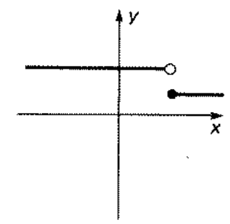
a.



b.



c.



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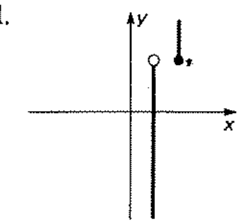
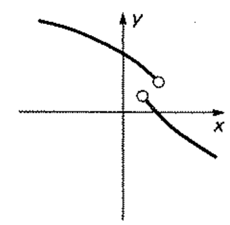


e.



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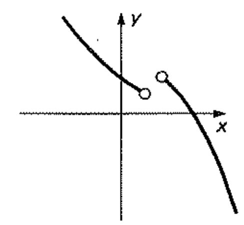

R3. For each of the following, sketch a reasonable graph showing how the *dependent* variable is related to the *independent* variable.

a. Your car stalls, and you roll to a stop without putting on the brakes. The car's speed depends on how long it has been since it stalled.

b. The length of time an oven has been turned on and the oven's temperature are related.
c. The angle at which you have to look up to see the Sun depends on the time of day. Make the domain extend for *several* days.
d. The number of cars in the student parking lot and the time of day are related.

CONCEPTS TEST

The following problems combine all of the objectives of this chapter. For each part of each problem, do what is asked, then identify by number which one or ones of the objectives you used in working that part.

T1. Consider the relation whose equation is

$$y = 1 + 4x - x^2.$$

a. Plot the graph of this relation, assuming that the domain is the set of non-negative numbers, and that the range must also contain only non-negative numbers.
b. What is the range of this relation?
c. Is the relation a function? Justify your answer.
d. At what value of y does the graph touch the y-axis?
e. At approximately what value of x does the graph touch the x-axis?
f. Tell *two* different real-world situations having graphs looking like this one.

T2. *Introduction to Polynomial Functions* A function is called a "polynomial function" if it has an equation of the form

$$y = \text{a polynomial involving } x.$$

a. Which of the relations in Problems 1–12 of Exercise 2-2 are polynomial functions?
b. Explain how closure insures that the domain of a polynomial function can be the set of *all* real numbers.
c. Explain how closure insures that a polynomial function really *is* a function.