Solution:
The easiest way to work this problem is to realize that horizontal lines have equations of the form y = constant. So you just write

$$\underline{y = 8.}$$

The graph is shown in Figure 3-4b.

The problem can also be worked by realizing that the slope of a horizontal line is 0. Using the point-slope form gives

$$y - 8 = 0(x - 7),$$

which can be transformed to $y = 8$.

EXAMPLE 5

Write the particular equation of the vertical line through (7, 8).

Solution:
The only way you can answer this question is to be brilliant, and just write down

$$\underline{x = 7.}$$

Since the slope of a vertical line does not equal a real number, neither the point-slope nor the slope-intercept form can be used. The graph is shown in Figure 3-4c.

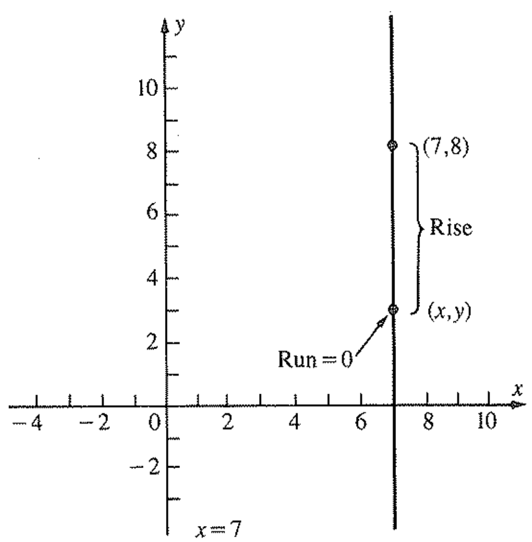


Figure 3-4c

In the following exercise you will get practice writing equations if information about the graph is given. It is this technique that will let you use linear functions in the next section to represent situations from the real world.

EXERCISE 3-4

Do These Quickly

The following problems are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. Write the general equation for slope-intercept form.

Q2. Find the x-intercept: $3x + 4y = 36$

Q3. Factor: $x^2 - x - 72$

Q4. Solve: $2x - 3 = 2(x + 4)$

Q5. 30 is 40% of what number?

Q6. Write the equation in the commutative axiom for addition.

Q7. Find the slope: $y = \frac{4}{7} + 3x$

Q8. Evaluate 5^3.

Q9. Divide $\frac{2}{3}$ by $\frac{3}{4}$.

Q10. Do the squaring: $(x - 3)^2$

Work these problems.

For Problems 1 through 26,

a. Write the particular equation of the line described.

b. Transform the equation (if necessary) to slope-intercept form.

c. Transform the equation to $Ax + By = C$ form, where A, B, and C are *integer* constants.

1. Has y-intercept of 21 and slope of -5.
2. Has y-intercept of -13 and slope of 7.
3. Contains (3, 7) and has a slope of 11.
4. Contains (4, 9) and has a slope of 5.73.
5. Contains (4, -5) and has a slope of -6.

6. Contains $(-3, 7)$ and has a slope of $-\frac{8}{5}$.
7. Contains $(1, 7)$ and $(3, 10)$.
8. Contains $(5, 2)$ and $(8, 11)$.
9. Contains $(2, -4)$ and $(-5, -10)$.
10. Contains $(-1, 4)$ and $(-5, -4)$.
11. Contains $(5, 8)$ and is parallel to the graph of $y = 7x - 6$.
12. Contains $(7, 2)$ and is parallel to the graph of $y = -4x + 3$.
13. Contains $(-4, 6)$ and is perpendicular to the graph of $y = 0.4x + 7$.
14. Contains $(3, -5)$ and is perpendicular to the graph of $y = -8x + 6$.
15. Contains $(5, 8)$ and is parallel to the graph of $2x + 3y = 9$.
16. Contains $(7, 2)$ and is parallel to the graph of $5x - 3y = 6$.
17. Contains $(4, 1)$ and is perpendicular to the graph of $5x - 7y = 44$.
18. Contains $(0, 6)$ and is perpendicular to the graph of $3x + 4y = 120$.
19. Has x-intercept of 5 and slope of $-\frac{2}{3}$.
20. Has x-intercept of 7 and y-intercept of 5.
21. Contains the origin, and has slope of 0.315.
22. Contains the origin, and has slope of 2.
23. Is horizontal, and contains $(-8, 9)$.
24. Is horizontal, and contains $(11, -13)$.
25. Is vertical, and contains $(-8, 9)$.
26. Is vertical, and contains $(11, -13)$.

For Problems 27 through 30, tell whether or not there is a linear function that contains *all* the points listed. If there is, find its particular equation.

27. $(6, 2), (5, 3), (1, 7)$
28. $(-3, 16), (1, 10), (9, -3)$
29. $(1, 4), (3, 7), (5, 10), (7, 13)$
30. $(4, 9), (20, 23), (13, 17), (29, 31)$
31. ***Intercept Form Problem*** Another form of the linear function equation is

$$\frac{x}{a} + \frac{y}{b} = 1,$$

where a and b stand for constants. Do the following.

a. Show that a and b are the x- and y-intercepts, respectively.
b. Transform the equation $\frac{x}{3} + \frac{y}{5} = 1$ to the following forms:
   i. $Ax + By = C$, where A, B, and C are integer constants.
   ii. $y = mx + b$.
c. Transform the equation $y = 4x - 12$ to the intercept form. Then tell what the two intercepts equal.
d. Why do you suppose the letter "b" is used in the slope-intercept form instead of some other letter?

32. ***Computer Program for Linear Function Equations***
   a. Write a computer program for finding the particular equation of a line from two given points. The input should be the two coordinates of the two points. The normal output should be an equation in slope-intercept form. The program should be able to do the proper thing when the slope is either zero or infinite.
   b. Test your program on the following pairs of points:
      i. (1, 7) and (3, 10).
      ii. (−1, −4) and (−5, 4).
      iii. (3, 8) and (6, 8).
      iv. (−2, 13), (−2, 4).
   c. Run your program with the ordered pairs (0, 3) and (2, 7). If it does not work, modify the program to get around the difficulty.

33. ***Computer Graphics Problem*** In this problem you will make some predictions about the graphs of various linear functions, then confirm (or refute!) your predictions by plotting the graphs on the computer. You may use the program PLOT LINEAR on the accompanying disk, or any other available plotting program.
   a. Without drawing the graph, how could you tell whether the graph of $3x + 5y = 30$ will go *up* or *down* as you go from left to right? Which way do you predict that it will go?
   b. Plot the graph of $3x + 5y = 30$ on the computer screen. Did the graph confirm your prediction in part (a)?
   c. How do you expect the graph of $3x + 5y = -20$ to be related to the graph in part (a)?
   d. Plot the graph of $3x + 5y = -20$ on the computer screen. Did the graph confirm your prediction in part (c)?
   e. What will be the slope of a graph perpendicular to the line in part (a)? What will be the equation of this perpendicular line if it has the same y-intercept as the line in part (c)?
   f. Plot the graph of the equation in part (e) on the screen. Is it really perpendicular to the other two? (You may have to adjust the vertical size on your monitor to make the scales on the axes the same before the lines will actually look perpendicular.)

34. ***Introduction to Linear Models*** Calvin Butterball drives from his home on the farm to the nearby town of Scorpion Gulch. As he

drives, his distance from Scorpion Gulch depends on the number of minutes he has been driving. When he has been driving for 6 minutes, he is 17 km away; when he has been driving for 15 minutes, he is 11 km away.

Let y be the number of kilometers Calvin is from Scorpion Gulch. Let x be the number of minutes Calvin has been driving.

a. Write the information about distances and times as two ordered pairs.
b. Plot the two ordered pairs on a Cartesian coordinate system.
c. Assume that the distance, time relation is a *linear* function. Draw the graph on the Cartesian coordinate system in part (b).
d. Write the particular equation for this function. Transform it, if necessary, so that y is by itself on one side of the equation.
e. Use the equation to predict Calvin's distance from Scorpion Gulch when he has been driving for 24 minutes.
f. Use the equation to predict the time when Calvin arrives at Scorpion Gulch.
g. In this problem, you have used a linear function as a "mathematical model." Why do you suppose these words are used?

3-5 LINEAR FUNCTIONS AS MATHEMATICAL MODELS

In Section 2-3 you drew "reasonable" graphs relating two real-world variables. Sometimes the graph was a straight line. You know how to find an equation for a linear function if information about the graph is given. This equation could be used to calculate values of one variable if values of the other variable are known. In this way the function can be used to predict things about the real world. A function used in this way is called a *mathematical model*.

Objective:

Given a situation in which two real-world variables are related by a straight-line graph, be able to:

a. Sketch the graph.
b. Find the particular equation.
c. Use the equation to predict values of either variable.
d. Figure out what the slope and intercepts tell you about the real world.

EXAMPLE 1

Driving Home Problem As you drive home from the football game, the number of kilometers you are away from home depends on the number of minutes you have been driving. Assume that distance varies linearly with time. Suppose that you are 11 km from home when you have been driving for 10 minutes, and 8 km from home when you have been driving for 15 minutes.

a. Define variables for distance and time, and sketch the graph.
b. Find the particular equation expressing distance in terms of time.
c. Predict your distance from home when you have been driving for 20, 25, and 30 minutes.
d. When were you 7 km from home?
e. What does the distance-intercept equal, and what does it represent in the real world?
f. What does the time-intercept equal, and what does it represent in the real world?
g. In what domain does this linear function give you reasonable answers?
h. What are the units of the slope? Based on these units, what do you suppose the slope represents in the real world? What is the significance of the fact that the slope is negative?

Solution:

a. Let d = no. of kilometers from home.
   Let t = no. of minutes you have been driving.
   The graph is shown in Figure 3-5a. All that is needed here is a reasonable sketch such as you drew in Section 2-3. Since you are assuming a linear function, the graph should be a straight line. It starts high and slopes downward since your distance from home decreases as you drive.

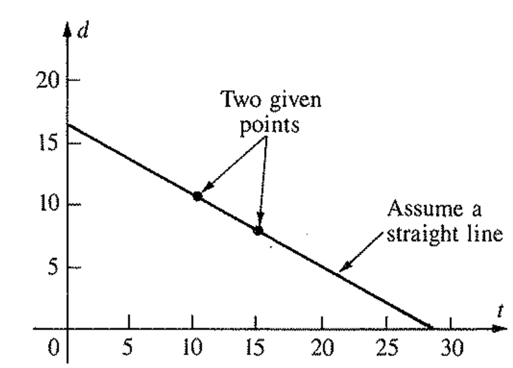


Figure 3-5a

b. To make this problem more familiar you can write the two given pieces of information as ordered pairs. Since d depends on t, you would write

(10, 11), (15, 8)

$$m = \frac{8 - 11}{15 - 10} = -\frac{3}{5}$$

Substituting the slope and the first ordered pair in the point-slope form gives

$$d - 11 = -\frac{3}{5}(t - 10)$$

The question calls for d to be expressed in terms of t. So you would transform it to slope-intercept form.

$$d = -\frac{3}{5}t + 17$$

c. To predict the distance when the time is given, all you need to do is substitute the given values for t and calculate d.

$$t = 20\text{: } d = -\frac{3}{5}(20) + 17 = 5 \text{ km}$$

$$t = 25\text{: } d = -\frac{3}{5}(25) + 17 = 2 \text{ km}$$

$$t = 30\text{: } d = -\frac{3}{5}(30) + 17 = -1 \text{ km}$$

Note that substituting 30 for t gives a negative value of distance. Since you would probably not drive past home, the domain of the function should stop before t reaches 30.

d. To predict the time when you are 7 km from home, you would substitute 7 for d and solve the equation.

$$7 = -\frac{3}{5}t + 17 \quad \text{Substitute 7 for } d.$$

$$\frac{3}{5}t = 10 \quad \text{Add } \frac{3}{5}t \text{ and subtract 7.}$$

$$t = \frac{50}{3} \quad \text{Multiply by } \frac{5}{3}.$$

about $16\frac{2}{3}$ *minutes*

e. The d-intercept is *17*, the value of d when $t = 0$. When $t = 0$ you are just starting for home. Therefore, it must be *17 km between the stadium and home.*

f. The t-intercept is the value of t when $d = 0$. Setting $d = 0$ in the equation gives

$$0 = -\frac{3}{5}t + 17$$
$$\frac{3}{5}t = 17$$
$$t = 28\frac{1}{3}$$

When $d = 0$ you are at home. So *it takes you about 28 $\frac{1}{3}$ minutes to get home*.

g. The domain should be $\{t: 0 \le t \le 28\frac{1}{3}\}$, the values of t for which you are actually driving home.

h. The slope is rise/run. The rise is in kilometers and the run is in minutes. Since "per" is a word used for "divided by," the slope has the units *km per min*. This means that your *speed is $\frac{3}{5}$ km/min*. The negative sign tells you that the distance from home is *decreasing* at $\frac{3}{5}$ km/min.

You should realize that the predictions you make with a mathematical model are no better than the assumptions you make in setting up the model. For the example above you assumed a *linear* function, that has a constant slope. If the speed varies, the graph would actually have different slopes at different places. Figure 3-5b shows what the actual graph might look like, and that the linear model may fit only approximately.

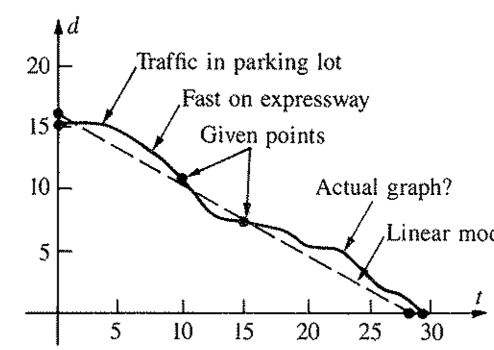


Figure 3-5b

Sometimes you are told a relationship between two variables which lets you conclude that a linear function relates them. In Example 2 you will see such a problem.

EXAMPLE 2

Donuts Problem

A local donut establishment charges 20 cents each for a donut, plus a one-time charge of 15 cents for the box, the service, etc.

a. Write an equation expressing the amount charged as a function of the number of donuts bought.
b. Explain why the function in part (a) is a *linear* function.
c. Predict the price of a box with a dozen donuts.
d. How many donuts would be in a box priced at $3.55?
e. Plot a graph of the function, using a reasonable domain.

Solution:

a. Let d = number of donuts in the box.
   Let p = number of cents you pay.
   The equation is $p = 20d + 15$.
b. The function is linear because the equation has the form p equals a linear (first degree) expression in the variable d.
c. $d = 12$: $p = 20(12) + 15 = 255$

   Answer: $2.55

   Note that 255 is the answer to the *algebra* problem. But the answer to the question that was asked is better written in the form of dollars, $2.55.
d. $p = 355$: $355 = 20d + 15$

   $$340 = 20d$$

   $$17 = d$$

   Answer: 17 donuts
e. The graph is shown in Figure 3-5c. Since you are asked to *plot* the graph, you must use graph paper, show scales, use a ruler, and so forth. The domain is just the non-negative integers since you seldom

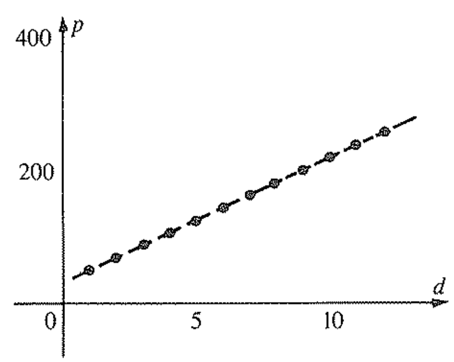


Figure 3-5c

buy fractions of donuts. The domain might stop at 12 if the store sells only boxes of up to a dozen. The p-intercept, 15, is excluded since you probably would not buy a box if you bought no donuts.

The following exercises will give you experience in using linear functions as mathematical models.

EXERCISE 3-5

Do These Quickly

The following problems are intended to refresh your skills. You should be able to do all 10 in less than 5 minutes.

Q1. Tell what axiom was used: $4(3 + x) = 4(x + 3)$

Q2. Find the slope: $3x + 5y = 30$

Q3. Simplify: $33 - 3(x + 7)$

Q4. Do the squaring: $(3x - 5)^2$

Q5. 35 is what percent of 50?

Q6. Write 0.375 as a fraction in lowest terms.

Q7. Draw the graph of a linear function with negative slope.

Q8. Draw the graph of a relation that is not a function.

Q9. Draw an isosceles triangle.

Q10. How many weeks are there in a year?

Work the following problems.

1. ***Computer Diskette Problem*** A computer store sells 10 floppy diskettes for \$15, and 30 diskettes for \$40. Assume that the number of dollars varies linearly with the number of diskettes. Write the particular equation expressing dollars in terms of diskettes, and use it to predict the price for a box of 100 diskettes. Sketch the graph.
2. ***Reading Problem*** Phoebe Small still has 35 pages of history to be read after she has been reading for 10 minutes, and 5 pages left after she has been reading for 50 minutes. Assume that the number of pages left to read varies linearly with the number of minutes she has been reading. Write the particular equation expressing pages in terms of minutes, and use it to predict the time when she has finished reading. Sketch the graph.
3. ***Milk Problem*** Handy Andy sells one-gallon cartons of milk (4 quarts) for \$3.09 each and half-gallon cartons for \$1.65 each. As-

sume that the number of cents you pay for a carton of milk varies linearly with the number of quarts the carton holds.
a. Write the particular equation expressing price in terms of quarts.
b. If Handy Andy sold 3-gallon cartons, what would your equation predict the price to be?
c. The actual prices for pint cartons ($\frac{1}{2}$ quart) and one-quart cartons are \$.57 and \$.99, respectively. Do these prices fit your mathematical model? If not, are they higher than predicted, or lower?
d. Suppose that you found cartons of milk marked at \$3.45, but that there was nothing on the carton to tell what size it is. According to your model, how much would such a carton hold?
e. Sketch the graph of the function, consistent with the slope and intercept in your equation.
f. What does the price-intercept represent in the real world?
g. What are the units of the slope? What real-world quantity does this number represent?

4. ***Reaction Time Problem*** When you are pricked with a pin, there is a short time delay before you say, "Ouch!" This reaction time varies linearly with the distance between your brain and the place you are pricked. Dr. Hollers pricks Leslie Morley's finger and toe, and measures reaction times of 15.2 and 22.9 milliseconds, respectively. (A millisecond is $\frac{1}{1000}$ of a second.) Leslie's finger is 100 cm from the brain, and her toe is 170 cm from the brain.
a. Write the particular equation expressing time delay in terms of distance.
b. How long would it take Leslie to say "Ouch!" if pricked in the neck, 10 cm from the brain?
c. What does the time-intercept equal, and what does it represent in the real world?
d. Sketch the graph of this function.
e. Since the slope is in milliseconds per centimeter, its reciprocal is the speed which nerve impulses travel in centimeters per millisecond. How fast do the impulses travel in cm/sec?

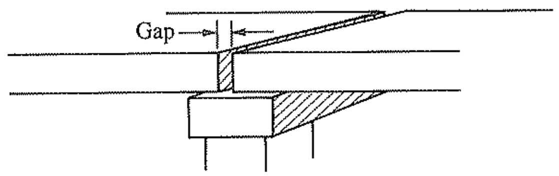
5. ***Cricket Problem*** Based on information in *Deep River Jim's Wilderness Trailbook*, the rate at which crickets chirp is a linear function of temperature. At 59°F they make 76 chirps per minute, and at 65°F they make 100 chirps per minute.
a. Write the particular equation expressing chirping rate in terms of temperature.
b. Predict the chirping rate for 90°F.
c. How warm is it if you count 120 chirps per minute?
d. Calculate the temperature-intercept. What does this number tell you about the real world?
e. Sketch the graph of this function in a reasonable domain.
f. What does the chirping-rate-intercept tell you about the real world?

6. *Cost of Owning a Car Problem* The number of dollars per month it costs you to own a car is a function of the number of kilometers per month you drive it. Based on information in an issue of *Time* magazine, the cost varies linearly with the distance, and is \$366 per month for 300 km per month, and \$510 per month for 1500 km per month.
   a. Write the particular equation expressing cost in terms of distance.
   b. Sketch the graph of this function.
   c. Predict your monthly cost if you drive 500, 1000, and 2000 km/month.
   d. About how far could you drive in a month without exceeding a monthly cost of \$600?
   e. What does the slope represent?
   f. List all the reasons you can think of to explain why the dollars per month intercept is greater than zero.

7. *Speed On a Hill Problem* Assume that the maximum speed your car will go is a linear function of the steepness of the hill it is going up or down. Suppose that the car can go a maximum of 55 mph up a 5° hill, and a maximum of 104 mph down a 2° hill. (Going downhill can be thought of as going up a hill of −2°.)
   a. Write the particular equation expressing maximum speed in terms of steepness.
   b. How fast could you go down a 7° hill?
   c. If your top speed is 83 mph, how steep is the hill? Is it up or down? Justify your answer.
   d. What does the speed-intercept equal, and what does it represent?
   e. What does the steepness-intercept equal, and what does it represent?
   f. Sketch the graph of this function, using a reasonable domain.

8. *Thermal Expansion Problem* Bridges on expressways often have expansion joints, which are small gaps in the roadway between one bridge section and the next. The gaps are put there so that the bridge will have room to expand when the weather gets hot. (See sketch.) Suppose that a bridge has a gap of 1.3 cm when the temperature is 22°C, and that the gap narrows to 0.9 cm when the temperature warms to 30°C. Assume that the gap width varies linearly with the temperature.
   a. Write the particular equation for gap width as a function of temperature.
   b. How wide would the gap be at 35°C? At −10°C?

c. At what temperature would the gap close completely? What mathematical name is given to this temperature?
d. Would the temperature ever be likely to get hot enough to close the gap? Justify your answer.
e. Sketch the graph of this linear function. Use an appropriate domain.



9. *Calorie Consumption Problem* H. E. Lansburg's *Weather and Health* reports data gathered during World War II which shows that people use about 30 more calories per day for each 1° drop in the Celsius temperature. At 21°C, a working person uses about 3000 calories per day.
   a. Explain how you know that the calorie consumption varies *linearly* with temperature. What is the slope? Write the particular equation.
   b. How many calories per day would a working person use
      i. in the Sahara Desert, when the temperature is 50°C?
      ii. in Antarctica in August, when the temperature is −50°C?
   c. At what temperature does your model predict that a working person would use no calories at all? Do you think the linear function gives meaningful answers for temperatures this hot? Explain.
   d. Sketch the graph of this function in a reasonable domain.

10. *Gas Tank Problem* Suppose that you get your car's gas tank filled up, then drive off down the highway. As you drive, the number of minutes, t, since you had the tank filled, and the number of liters, g, remaining in the tank are related by a linear function.
   a. Which variable should be independent, and which should be dependent?
   b. After 40 minutes you have 52 liters left. An hour after the fill-up you have 40 liters left. Write the particular equation for this function.
   c. Use the equation to predict the time when you will run out of gas.
   d. Find the g-intercept, and tell what it represents in the real world.
   e. Sketch the graph of this linear function.
   f. Tell what the slope represents in the real world, and tell the significance of the fact that the slope is negative.

11. *Terminal Velocity Problem* If you jump out of an airplane at high altitude, but do not open your parachute, you will soon be falling at a constant velocity called your "terminal velocity." Suppose that at time $t = 0$ you jump. When $t = 15$ seconds, your wrist altimeter shows that your distance from the ground, d, is 3600 meters. When $t = 35$, you have dropped to $d = 2400$ meters. Assume that you have already reached your terminal velocity by the time $t = 15$.
 a. Explain why d varies *linearly* with t after you have reached your terminal velocity.
 b. Write the particular equation expressing d in terms of t.
 c. If you neglect to open your parachute, when will you hit the ground?
 d. According to your linear model, how high was the airplane when you jumped?
 e. The airplane was actually at 4200 meters when you jumped. How do you reconcile this fact with your answer to part (d)?
 f. Sketch a reasonable graph of d versus t, showing the linear part, the part before you reached terminal velocity, and the part after you open your parachute.
 g. What was your terminal velocity in meters per second? In kilometers per hour?

12. *Linear Depreciation Problem* Suppose you own a car that is presently 40 months old. From an automobile dealer's "Blue Book" you find that its present trade-in value is $3300. From an old Blue Book you find that its trade-in value 10 months ago was $4700. Assume that its trade-in value decreases linearly with time.
 a. Write the particular equation expressing trade-in value of your car as a function of its age in months.
 b. You plan to get rid of the car when its trade-in value drops to $1000. How much longer can you keep the car?
 c. By how many dollars does the car "depreciate" (decrease in value) each month? What part of the mathematical model tells you this?
 d. When do you predict that the car will be worthless? What part of the mathematical model tells you this?
 e. According to your linear model, what was the trade-in value of your car when it was new?
 f. The car actually cost $10,560 when it was new. How do you explain the difference between this number and the answer to part (e)?
 g. Sketch the graph of this function.

13. *Shoe Size Problem* The size of shoe a person needs varies linearly with the length of his or her foot. The smallest adult shoe is Size 5, and fits a 9-inch long foot. An 11-inch long foot takes a Size 11 shoe.

a. Write the particular equation expressing shoe size in terms of foot length.
b. If your foot is a foot long, what size shoe do you need?
c. Bob Lanier, who once played basketball for the Detroit Pistons, wears a Size 22 shoe. How long is his foot?
d. Plot the graph of adult shoe size versus foot length. Be sure to observe the domain implied at the beginning of this problem.

14. *Hippopotamus Problem* In order to hunt hippopotami, a hunter must have a hippopotamus hunting license. Since the hunter can sell the hippos he catches, he can use the proceeds to pay for part or all of the cost of the license. If he catches only 3 hippos, he is still in debt by \$2050. If he catches 7 hippos, he makes a profit of \$1550. The African Game and Wildlife Commission allows a limit of 10 hippos per hunter. Let h be the number of hippos caught, and let d be the number of dollars profit made. Assume that h and d are related by a linear function.
a. Which variable should be dependent, and which should be independent?
b. Write a suitable domain for the independent variable.
c. Write the particular equation expressing dependent variable in terms of independent variable.
d. Plot the graph of this function, observing the domain in part (b).
e. Calculate the d- and h-intercepts. Tell what each means in the real world.
f. State what real-world quantity the slope represents.
g. What is the origin of the word "hippopotamus?"

15. *Celsius-to-Fahrenheit Temperature Conversion* The Fahrenheit temperature, "F," and the Celsius temperature, "C," of an object are related by a linear function. Water boils at 100°C or 212°F, and freezes at 0°C or 32°F.
a. Write an equation expressing F in terms of C.
b. Transform the equation so that C is in terms of F.
c. Lead boils at 1620°C. What Fahrenheit temperature is this? Which form of the equation is more appropriate to use in answering this question?
d. Normal body temperature is 98.6°F. What Celsius temperature is this?
e. If the weather forecaster says it will be 40°C today, will it be hot, cold, or medium? Explain.
f. The coldest possible temperature is absolute zero, −273°C, where molecules stop moving. What Fahrenheit temperature is this?
g. For what temperature is the number of Fahrenheit degrees equal to the number of Celsius degrees?
h. Sketch the graph of F as a function of C, showing clearly the domain implied by part (f), and the F-intercept.

16. *Charles's Gas Law* In 1787 the French scientist Jacques Charles observed that when he plotted the graph of volume of a fixed amount of air versus the temperature of the air, the points lay along a straight line. Therefore, he concluded that volume of a gas varies linearly with temperature. Suppose that at 27°C a certain amount of air occupies a volume of 500 cm^3. When it is warmed to 90°C it occupies 605 cm^3.
    a. Write the particular equation expressing volume in terms of temperature.
    b. Predict the volume at 60°C.
    c. The process of predicting a value *between* two given data points is called "interpolation." What is the origin of this word?
    d. Predict the volume at 300°C.
    e. The process of predicting a value *beyond* any given data points is called "extrapolation." What is the origin of this word?
    f. Extrapolate your mathematical model back to the point where the volume is zero. That is, find the temperature-intercept.
    g. Find out what special name is given to the temperature in part (f).
    h. Sketch the graph of volume versus temperature.

17. *Elevator Problem* The number of feet of cable needed for an elevator depends on the number of stories in the building it serves. Suppose that $c = 20s + 35$, where c is the number of feet of elevator cable and s is the number of stories.
    a. How do you know that c varies *linearly* with s?
    b. How much cable is needed for a 29-story building?
    c. How tall a building needs 375 feet of cable?
    d. What does the slope represent in the real world?
    e. What does the c-intercept equal? Why do you suppose that it is greater than zero?
    f. Write a suitable domain for the linear function.
    g. On graph paper, plot the graph of this function, observing the domain you wrote in part (f).

18. *Speeding Bullet Problem* The speed a bullet is traveling depends on the number of feet the bullet has traveled since it left the gun. Assume that $s = -4d + 3600$, where s is the number of feet per second and d is the number of feet.
    a. How do you know that s varies *linearly* with d?
    b. How fast is the bullet going when it has traveled 300 feet?
    c. How far has the bullet gone when it has slowed to 500 feet per second?
    d. What does the slope represent in the real world?
    e. What does the d-intercept equal? What does it tell you about the bullet?
    f. Write a suitable domain for the linear function.

g. On graph paper, plot the graph of this function, observing the domain you wrote in part (f).

19. ***Income Tax Problem*** Starting in 1988, the Internal Revenue Service's formula for finding income tax was \$9,761 plus 33% of the quantity (Taxable Income minus \$43,150). This formula applied for taxable incomes between \$43,150 and \$89,560. Let x be number of dollars of taxable income.
   a. Write the particular equation expressing tax in terms of taxable income.
   b. Find the tax for a person making \$43,150, and for another person making \$89,560.
   c. Show that the tax for a person making \$89,560 is exactly 28% of his or her taxable income.
   d. Plot the graph of this function in the domain for which it applies.
   e. For taxable incomes above \$89,650, the formula for tax due is simply 28% of the taxable income. Calculate the tax on a taxable income of \$120,000.
   f. Plot the graph of the tax for incomes above \$89,560 on your graph in part (d). If your work is right, the two graphs should connect.

20. ***Direct Variation, Pancake Problem*** If the constant b in $y = mx + b$ equals zero, then y is said to vary *directly* with x. The amount of pancake batter you must mix up varies directly with the number of people who come to breakfast. Suppose that it takes 7 cups of batter to serve 10 people.
   a. Write the particular equation expressing number of cups in terms of number of people.
   b. How many cups must you prepare for 50 people?
   c. About how many people can you serve with 12 cups of batter?
   d. Sketch the graph of this function. Through what special point does the graph of a direct variation function go?

3-6 CHAPTER REVIEW AND TEST

In this chapter you have studied one kind of function, the linear function. You defined it by the general equation $y = mx + b$. You graphed it, first by pointwise plotting. Then you learned properties that allowed you to plot the graph quickly. Finally, you learned how to find the particular equation from given points so that you could use a linear function as a mathematical model.

The Review Problems below parallel the sections in this chapter. The Concepts Problems let you try your hand at applying what you know to analyze a new situation. The Chapter Test is similar to one your instructor might give to see how well you understand linear functions.

REVIEW PROBLEMS

R1. Given the equation $y = 3x - 7$:

a. Evaluate y if $x = -1$, $x = 2$, and $x = 5$.

b. Show by graphing that the points lie in a straight line.

R2. Plot quickly the graphs of the following:

a. $y = \frac{2}{5}x - 3$

b. $7x + 3y = 21$

c. $x = 3$

d. $y = -4$

R3. For the linear function $y + 3 = -\frac{5}{2}(x - 4)$

a. Name the form of the equation.

b. Write the coordinates of the point that appears in the equation.

c. Plot the graph.

d. Transform the equation to slope-intercept form.

e. Transform the equation to $Ax + By = C$ form, where A, B, and C are integers.

R4. Find the particular equation of the line described.

a. Contains $(2, -7)$ and $(5, 3)$.

b. Contains $(-4, 1)$ and is parallel to the graph of $2x - 9y = 47$.

c. Contains $(3, -8)$ and is perpendicular to the graph of $y = 0.2x + 11$

d. Has x-intercept of 5 and y-intercept of -6.

e. Is vertical, and contains $(-13, 8)$.

f. Is horizontal, and contains $(22, \pi)$.

g. Has the x-axis as its graph.

h. Has a slope that is infinitely large, and contains $(5, 7)$.

R5. *Computer Time Problem* If a computer program has a loop in it, the length of time it takes the computer to run the program varies linearly with the number of times it must go through the loop. Suppose a computer takes 8 seconds to run a given program when it goes through the loop 100 times, and 62 seconds when it loops 1000 times.

a. Write the particular equation expressing seconds in terms of loops.

b. Predict the length of time needed to loop 30 times; 10,000 times.

c. Suppose the computer takes 23 seconds to run the program. How many times does it go through the loop?
d. How long does it take the computer to run the rest of the program, excluding the loop? What part of the mathematical model tells you this?
e. How long does it take the computer to go through the loop once? What part of the mathematical model tells you this?
f. Plot the graph of this function.

CONCEPTS PROBLEM

Income Tax Problem The amount of income tax you pay to the Federal Government varies linearly with the amount of taxable income you have. However, the percent of your income that you pay in tax is different in different parts of the domain. For 1988 the rates for single taxpayers were specified the following way by the Internal Revenue Service:

Taxable Income (TI)	Tax Payable
\$0 to \$17,850	15% of TI
\$17,850 to \$43,150...	\$2,677.50 plus 28% of (TI − \$17,850)
\$43,150 to \$89,560...	\$9,761.50 plus 33% of (TI − \$43,150)
\$89,560, up	28% of TI.

a. Write four particular equations expressing tax payable in terms of taxable income, one for each part of the domain.
b. What will be the tax for \$10,000? \$30,000? \$50,000? \$100,000?
c. You overhear somebody say they paid \$20,000 in taxes. What was their taxable income?
d. Show that there are no discontinuities at the three places where the tax rate changes. You must figure out what "discontinuities" means, and how to answer this question.
e. Draw the graph of tax payable versus taxable income for incomes from \$0 through \$100,000.
f. A \$2 change in income, from \$17,849 to \$17,851, would put a person in a higher "tax bracket." That is, one would be paying 28% instead of 15%. Many people think that the extra \$2 earned would cause them to pay hundreds of dollars more in taxes. Is their thinking correct? Explain.
g. Faye Doubt makes \$60,000 taxable income. She is upset because she is in a 33% tax bracket, while people who make \$100,000 are in only a 28% tax bracket. Convince Faye that she would pay *more* tax on her \$60,000 using the equation for the higher-income person.
h. The Internal Revenue Service rules for the two middle-income tax brackets look a lot like one of the forms of the linear function equation. Which form?

CHAPTER TEST

Algebra II is a study of functions. The first kind of function you have studied is the *linear* function.

T1. Write the general equation for a linear function.

For Problems T2 and T3, transform to slope-intercept form.

T2. $3x - 7y = 42$

T3. $y - 5 = -\frac{1}{2}(x - 3)$

T4. Write the definition of x-intercept. Use this definition to find the x-intercept for the function whose equation is $y = 5x - 7$.

T5. ***Beans Problem*** Handy Andy sells 23 oz cans of ranch style beans for 63 cents and 15 oz cans for 45 cents. Assume that the price varies linearly with the number of ounces.

a. Write the particluar equation expressing number of cents in terms of number of ounces.
b. Sketch the graph.
c. A 52 oz can costs $1.39. According to your model, is this can over-priced or under-priced? By how much?
d. Suppose that an "individual serving" can was priced at $0.21. About how many ounces of beans would you expect to get?

It is important for you to remember old techniques.

T6. Tell the range and domain of the relation in Figure 3-6a, and whether or not it is a function.

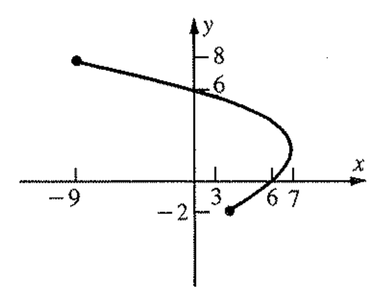


Figure 3-6a

T7. The length of your thumbnail depends on how long it has been since you cut the nail. Sketch a reasonable graph.