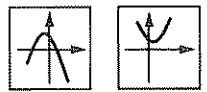


- e. Draw a graph of the function in the appropriate domain.
- f. Why is a quadratic function more reasonable for this problem than a linear function would be?
3. *Car Insurance Problem* Suppose that you are an actuary for F. Bender's Insurance Agency. Your company plans to offer a senior citizen's accident policy, and you must predict the likelihood of an accident as a function of the driver's age. From previous accident records, you find the following information:

Age (years)	Accidents per 100 Million Kilometers Driven
20	440
30	280
40	200

You know that the number of accidents per 100 million kilometers driven should reach a minimum then go up again for very old drivers. Therefore, you assume that a *quadratic* function is a reasonable model.

- a. Write the particular equation expressing accidents per 100 million kilometers in terms of age.
- b. How many accidents per 100 million kilometers would you expect for an 80-year-old driver?
- c. Based on your model, who is safer; a 16-year-old driver or a 70-year-old driver?
- d. What age driver appears to be the safest?
- e. Your company decides to insure licensed drivers up to the age where the accident rate reaches 830 per million kilometers. What, then, is the domain of this quadratic function?
4. *Cost of Operating a Car Problem* The number of cents per kilometer it costs to drive a car depends on how fast you drive it. At low speeds the cost is high because the engine operates inefficiently, while at high speeds the cost is high because the engine must overcome high wind resistance. At moderate speeds the cost reaches a minimum. Assume, therefore, that the number of cents per kilometer varies *quadratically* with the number of kilometers per hours (kph).
- a. Suppose that it costs 28, 21, and 16 cents per kilometer to drive at 10, 20, and 30 kph, respectively. Write the particular equation for this function.
- b. How much would you spend to drive at 150 kph?
- c. Between what two speeds must you drive to keep your cost no more than 13 cents per kilometer?
- d. Is it possible to spend only 10 cents per kilometer? Justify your answer.
- e. The *least* number of cents per kilometer occurs when you get the *most* kilometers per liter of gas. If your tank were nearly empty, at what speed should you drive to have the best chance of making it to a gas station before you run out?



5. **Artillery Problem** Artillerymen on a hillside are trying to hit a target behind a mountain on the other side of the river (see Figure 5-8c). Their cannon is at  $(x, y) = (3, 250)$ , where  $x$  is in kilometers and  $y$  is in meters. The target is at  $(x, y) = (-2, 50)$ . In order to avoid hitting the mountain on the other side of the river, the projectile from the cannon must go through the point  $(x, y) = (-1, 410)$ .

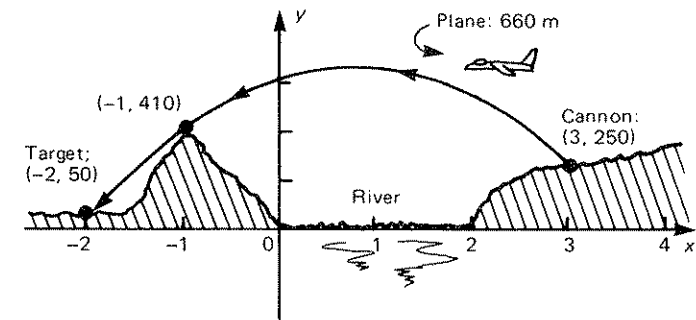


Figure 5-7c

- Write the particular equation of the parabolic path of the projectile.
  - How high above the river will the projectile be where it crosses
    - the right riverbank,  $x = 2$ ?
    - the left riverbank,  $x = 0$ ?
  - Approximately where will the projectile be when  $y = 130$ ?
  - A reconnaissance plane is flying at 660 meters above the river. Is it in danger of being hit by projectiles fired along this parabolic path? Justify your answer.
6. **Football Problem** When a football is punted, it goes up into the air, reaches a maximum altitude, then comes back down. Assume, therefore, that a quadratic function is a reasonable mathematical model for this real-world situation.

Let  $t$  = number of seconds that have elapsed since the ball was punted.

Let  $d$  = number of feet the ball is above the ground.

- When the ball was kicked it was 4 feet above the ground. One second later, it was 28 feet above the ground. Two seconds after it was kicked, it was 20 feet up. Write the particular equation expressing  $d$  in terms of  $t$ .
- Find the  $d$  and  $t$  coordinates of the vertex, and tell what each represents in the real world.
- Find the  $t$ -intercepts and tell what each represents in the real world. Use square root tables or an educated guess to find a decimal approximation for any square roots you may encounter.

- d. Draw a graph of the function. Select scales which make the graph fill up most of the sheet of graph paper. You should have enough points with the vertex, intercepts, given points, and points obtainable easily from symmetry.
- e. By looking at your graph and thinking about what it represents, figure out a *domain* for this function. Why would your model not give reasonable answers for  $d$  when the value of  $t$  is
  - i. below the lower bound of the domain?
  - ii. above the upper bound of the domain?
 Modify your graph, if necessary, to agree with this domain.
- f. What influences in the real world might make your model slightly inaccurate *within* the domain?
- g. From your graph and your calculations, tell what the *range* of this function is.

7. **Rectangular Field Problem** A rectangular field is 300 yards by 500 yards. A roadway of width  $x$  yards is to be built inside the field (see Figure 5-7d). This problem concerns the region inside the roadway.

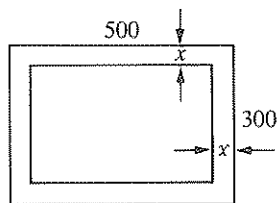


Figure 5-7d

- a. Write the length and width of the region as functions of  $x$ . What kind of functions are these?
  - b. Write the area of the region as a function of  $x$ . What kind of function is this?
  - c. Predict the area of the region if  $x = 5, 10,$  and  $15$ .
  - d. What is the widest the roadway can be and still leave 100,000 square yards in the region?
  - e. Sketch the graph of area versus  $x$  in a reasonable domain.
  - f. Find the value of  $x$  which makes the roadway have an area equal to the area of the field.
8. **Corral Problem** A rectangular corral is to be built by stringing an electric fence as shown with  $y$  feet for the side parallel to the river and  $x$  feet for each of the two sides perpendicular to the river (Figure 5-7e). The total length of the fence is to be 900 feet.
- a. Write an equation expressing  $y$  in terms of  $x$ . What kind of function is this?
  - b. Let  $A(x)$  be the number of square feet area taken up by the corral. Write the particular equation for  $A(x)$ . What kind of function is this?

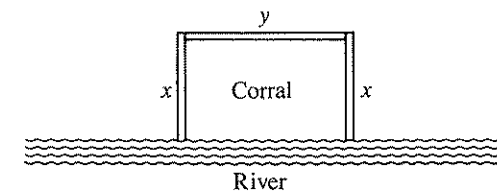
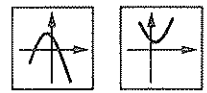


Figure 5-7e

- c. Find  $A(100)$  and  $A(300)$ .
  - d. What value of  $x$  makes  $A(x)$  a maximum? What is this maximum area?
  - e. What values of  $x$  make  $A(x)$  equal 0?
  - f. Sketch the graph of function  $A$  in a reasonable domain.
9. *Loan Problem* You borrow some money from your parents to buy a used motorcycle, and agree to pay it off at \$17 a week. After 13 weeks you still owe them \$544. Answer the following questions:
- a. Write the particular equation expressing the amount you still owe in terms of the number of weeks.
  - b. Explain why the amount you still owe is a *linear* function of the number of weeks.
  - c. How much will you still owe after 20 weeks?
  - d. When will the amount you owe first drop below \$100?
  - e. Find the dollars intercept. What does this represent?
  - f. Find the weeks intercept. What does this represent?
  - g. Sketch the graph, using a suitable domain.
10. *Breathing Problem* Your nose, windpipe, and so forth, hold about a pint of air. So when you breathe in, the first pint of air to reach your lungs is air you have breathed before. If you breathe more than a pint, the rest of the air reaching your lungs is fresh air. The maximum amount you can inhale on any one breath is about 4 pints.
- a. Sketch a graph of  $f$ , the pints of fresh air reaching the lungs, as a function of  $a$ , the total pints of air breathed in. Explain why the function is linear, and tell its slope and  $a$ -intercept.
  - b. Write the particular equation expressing  $f$  in terms of  $a$ .
  - c. What percent of the air reaching your lungs is fresh air if you inhale 1 pint? 2 pints? 3 pints? 4 pints?
  - d. Plot the graph of percent of fresh air reaching the lungs as a function of pints of air breathed in. Is the function linear? Justify your answer.
  - e. What would happen to you if you breathed very shallow breaths, less than a pint, for a long period of time?
11. *Dee Side's Pig Problem* Dee Side has a pig that presently weighs 200 pounds. She could sell it now for a price of \$1.40 a pound.
- a. What is the worth of the pig now?

- b. The pig is gaining 5 pounds a week. Write an equation for its weight as a function of weeks. What kind of function is this?
- c. The price per pound is dropping 2 cents a week. Write an equation for price per pound as a function of weeks. What kind of function is this?
- d. Write an equation for the total worth of the pig as a function of weeks. What kind of function is this?
- e. Predict the worth of the pig after 8 weeks, 16 weeks, and 24 weeks.
- f. When should Dee sell the pig to get the maximum amount of money for it?
- g. Sketch the graph of worth versus time in a reasonable domain.
12. *Luke and Leia Problem* Luke and Leia are trapped in a room on a space station. The room is 20 meters long and 15 meters wide. But the length is decreasing linearly with time at a rate of 2 meters per minute, and the width is increasing linearly with time at a rate of 3 meters per minute.
- a. Let  $L(t)$  and  $W(t)$  be the length and width of the room, respectively, in meters. Let  $t$  be the number of minutes since the room was 20 by 15. Write particular equations for functions  $L$  and  $W$ .
- b. Let  $A(t)$  be the number of square meters of floor area in the room. Write the particular equation for function  $A$ .
- c. Does the area of the room reach a maximum for a positive value of  $t$ ? If so, what value of  $t$ ? If not, how do you tell?
- d. When will the area of the room be zero?
- e. Sketch the graph of area versus time.
13. *Quadratic or Linear Problem Number 1* In science lab, Wanda Ngo measures the following values of  $x$  and  $y$ :

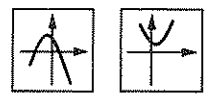
$\frac{x}{11}$	$\frac{y}{394}$
18	807
23	471

Wanda wants to know whether the function could be linear. Tell her the answer, and how you decided. If it is linear, find the particular equation. If it is not linear, assume it is quadratic and find the particular equation.

14. *Quadratic or Linear Problem Number 2* In science lab, Ida Ngo measures the following values of  $x$  and  $y$ :

$\frac{x}{11}$	$\frac{y}{394}$
18	807
23	1162

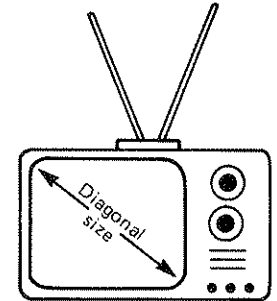
Ida wants to know whether the function could be linear. Tell her the



answer, and how you decided. If it is linear, find the particular equation. If it is not linear, assume it is quadratic and find the particular equation.

15. **Color TV Problem** The following are prices of a popular brand of color TV:

Size (in.)	Price
5"	\$450
9"	\$430
12"	\$400
15"	\$450
17"	\$510
19"	\$570
21"	\$700



- Plot these ordered pairs (size, price) on a Cartesian coordinate system. Connect the dots with a smooth curve.
  - Assume that the price varies quadratically with screen size. Use the ordered pairs of 9", 15", and 19" screens to derive the particular equation for this function.
  - If the manufacturer produced a 24" TV set, how much would you expect to pay for one?
  - Use the equation to calculate the prices of 5", 12", 17", and 21" diagonal sets.
  - Plot the predicted points from part d on the Cartesian coordinate system of part a. Connect these points with a smooth curve. Use means such as a colored pencil to distinguish clearly between the two graphs.
  - Based on the two graphs, would you describe the quadratic model as *accurate*, *reasonable*, or *inaccurate*?
  - Why do you suppose the price goes *up* as the size gets very small?
16. **Calvin Butterball's Gasoline Problem** Calvin Butterball is driving along the highway. He starts up a long, straight hill (see Figure 5-7f). 114 meters from the bottom of the hill Calvin's car runs out of gas. He doesn't put on the brakes, so the car keeps rolling for awhile, coasts to a stop, then starts rolling backwards. He finds that his distances from the bottom of the hill 4 and 6 seconds after he runs out of gas are 198 and 234 meters, respectively.
- There are *three* ordered pairs of time and distance in the information given above. What are they?
  - Write an equation expressing Calvin's distance from the bottom of the hill in terms of the number of seconds which have elapsed since he ran out of gas. Assume that a quadratic function is a reasonable mathematical model.

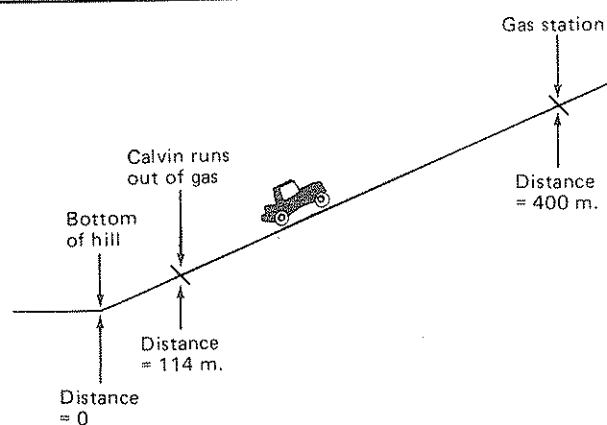


Figure 5-7f

- c. Use the model to predict Calvin's distance from the bottom of the hill 10 and 30 seconds after he ran out of gas.
  - d. Draw a graph of this function. Use the given points, the calculated points from part c, and any other points you find useful.
  - e. Last Chance Texaco Station is located 400 meters from the bottom of the hill. Based on your model, will Calvin's car reach the station before it stops and starts rolling backwards? Justify your answer.
17. *Spaceship Problem* When a spaceship is sent to the Moon, it is first put into orbit around the Earth. Then just at the right time, the rocket motor is fired to start it on its parabolic path to the Moon (see Figure 5-7g). Let  $x$  and  $y$  be coordinates measuring the position of

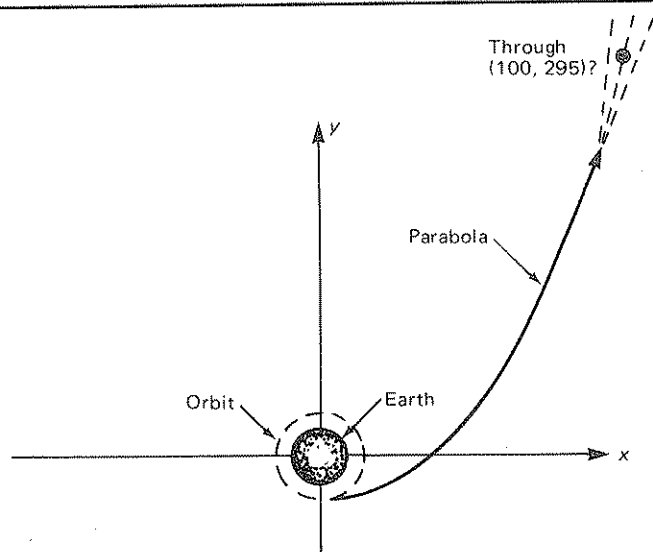


Figure 5-7g



the spacecraft (in thousands of kilometers) as it goes Moonward. When the rocket is fired,  $x = 0$  and  $y = -7$ . The tracking station measures the position at two later times, and finds that  $y = -4$  when  $x = 10$ , and  $y = 5$  when  $x = 20$ .

- a. Find an equation of the parabolic path in Figure 5-7g.
  - b. In order to hit the Moon without a mid-course maneuver, the path of the spaceship must pass through the point  $(100, 295)$ . Use your mathematical model to predict whether or not a mid-course maneuver will be necessary.
18. **Gateway Arch Problem** On a trip to St. Louis you visit the Gateway Arch. Since you have plenty of time on your hands, you decide to estimate its altitude. You set up a Cartesian coordinate system with one end of the arch at the origin, as shown in Figure 5-7h. The other end of the arch is at  $x = 162$  meters. To find a third point on the arch, you measure a value of  $y = 4.55$  meters when  $x = 1$  meter. You assume that the arch is parabolic.

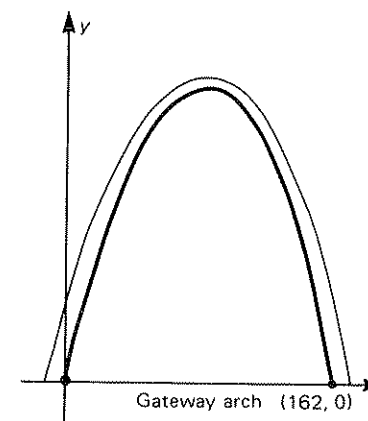


Figure 5-7h

- a. Find the particular equation of the underside of the arch.
  - b. What is the  $x$ -coordinate of the vertex? By substituting this number into the equation, predict the height of the arch.
  - c. An airplane with a wingspan of 40 meters tries to fly through the arch at an altitude of 170 meters. Could the plane possibly make it? Justify your answer.
19. **Barley Problem** The number of bushels of barley an acre of land will yield depends on how many seeds per acre you plant. From previous planting statistics you find that if you plant 2 hundred thousand seeds per acre, you can harvest 22 bushels per acre, and if you plant 4 hundred thousand seeds per acre you can harvest 40 bushels per acre. As you plant more seeds per acre, the harvest will reach a max-



imum, then decrease. This happens because the young plants crowd each other out and compete for food and sunlight. Assume, therefore, that the number of bushels per acre you can harvest varies *quadratically* with the number of millions of seeds per acre you plant.

- Write *three* ordered pairs of (millions of seeds, bushels). The third ordered pair is not given above, but should be obvious.
- Write the particular equation for this function.
- How many bushels per acre would you expect to get if you plant 16 hundred thousand seeds per acre?
- Based on your model, would it be possible to get a harvest of 70 bushels per acre? Justify your answer.
- How much should you plant to get the *maximum* number of bushels per acre?
- According to your model, is it possible to plant so many seeds that you harvest no barley at all?
- Plot the graph of this quadratic function in a suitable domain.



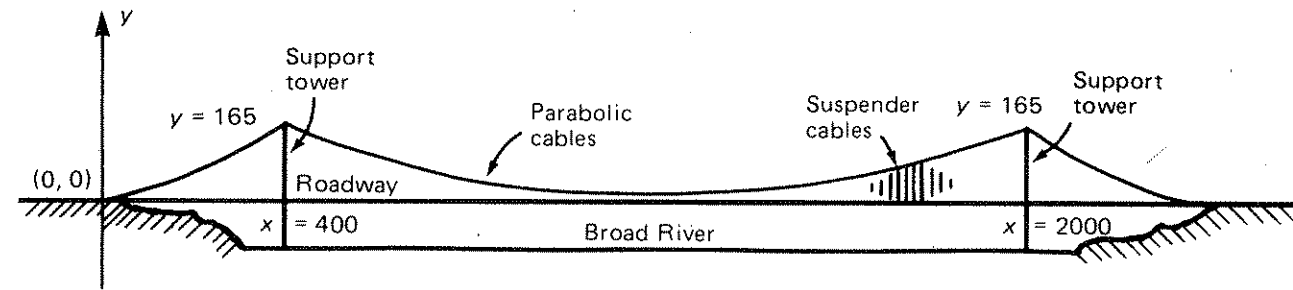
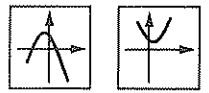


Figure 5-7i

20. **Suspension Bridge Problem** In a suspension bridge such as the Golden Gate Bridge in California or the Verrazano-Narrows Bridge in New York, the roadway is supported by parabolic cables hanging from support towers, as shown in Figure 5-7i. Vertical suspender cables connect the parabolic cables to the roadway.

Suppose that you work for Ornery & Sly Construction Company. Your job is to procure the vertical suspender cables for the center span of the new bridge to be built across Broad River. From the Design Department, you find the following information:

- i. The support towers are located at  $x = 400$  meters and  $x = 2000$  meters (making this span slightly longer than the Verrazano-Narrows Bridge). The tops of the towers are 165 meters above the roadway, at  $y = 165$ .
  - ii. There are four parallel parabolic cables that go from tower top to tower top. They are 5 meters above the roadway at their lowest points, halfway between the two towers.
  - iii. The vertical suspender cables are spaced every 20 meters, starting at  $x = 420$  and ending at  $x = 1980$ . Each of the four parabolic cables has *two* suspender cables at each value of  $x$ .
- a. Demonstrate that you understand the above information by writing *three* ordered pairs for points on the parabolic cables.
  - b. Find the particular equation of the parabolic cables. You may use the vertex form or the  $y = ax^2 + bx + c$  form, whichever seems more convenient.
  - c. The success of the \$700,000,000 project (and whether or not you keep your job!) depends on the correctness of your calculations. Demonstrate that your equation gives correct values of  $y$  by substituting the three values of  $x$  from the given ordered pairs in part a.
  - d. As a further check on the correctness of your equation, substitute 0 for  $y$ , and show that the resulting values of  $x$  are consistent with the given information.

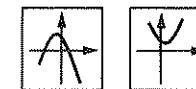
- e. Write a computer program to calculate and print the length of each vertical suspender cable and its corresponding value of  $x$ . As each value is calculated, the computer should add it to the sum of the previous values of  $y$ . When all values are printed, the computer should print the total length of suspender cable that will be used for the center span so that the Purchasing Department may buy the correct amount of cable.
- f. Remember, you will be *fired* if you present your boss with an incorrect answer. Perform a *quick* calculation to show that the total computed cable length is *reasonable*.

In this chapter you have studied the second major kind of function, the quadratic function. You have learned that the graphs are curved, and are called parabolas. While solving quadratic equations to find  $x$  for given values of  $y$ , you ran across imaginary and complex numbers. Finding the particular equation for mathematical models problems required that you solve a system of linear equations, as you did in Chapter 4. You also made progress in distinguishing between linear and quadratic model problems.

The Review Problems below parallel the sections in this chapter. The Concepts Problems let you try your hand at applying what you know to analyze a new situation. The Chapter Test is similar to one your instructor might give to see how well you understand quadratic functions.

#### REVIEW PROBLEMS

- R1. If  $f(x) = 3x^2 - 7x + 11$ , find  $f(-5)$ . Tell what kind of function  $f$  is.
- R2. a. Do the squaring:  $(3x - 4)^2$   
 b. Transform to vertex form:  $y = 5x^2 + 14x - 3$   
 c. Sketch a parabola with vertex at  $(5, 11)$  and  $y$ -intercept 3.  
 d. Find the  $x$ -intercepts:  $y = -6x^2 + 10x + 7$
- R3. a. Solve:  $8x^2 - 5x - 30 = 4$  (Watch out!)  
 b. Find the discriminant:  $9x^2 + 11x + 13 = 0$   
 c. Describe the solutions of the quadratic equation with rational coefficients if the discriminant equals:  
 i. 121  
 ii. 50  
 iii. 0  
 iv. -9



- d. Find the vertex *without* transforming to vertex form:  
 $y = 5x^2 + 17x + 91$
- R4. a. Write in terms of  $i$ :  $\sqrt{-64}$   
 b. Plot  $-3 + 2i$  on the complex plane.  
 c. Solve:  $3x^2 - 8x + 50 = 2x(x - 3)$
- R5. If  $g(x) = x^2 - 7x + 4$ ,  
 a. find  $g(9)$ ,  
 b. find  $x$  if  $g(x) = 42$ ,  
 c. tell whether or not  $g(x)$  ever equals  $-10$ .
- R6. a. Find the particular equation of the quadratic function containing  $(2, 1)$ ,  $(5, 31)$ , and  $(-1, 7)$ .  
 b. Find the particular equation of the linear function containing  $(7, 4)$  and  $(10, 19)$ .
- R7. **Diving Board Problem** Jack Potts dives off the high diving board. His distance from the surface of the water varies *quadratically* with the number of seconds that have passed since he left the board.  
 a. His distances at times of 1, 2 and 3 seconds since he left the board are 24, 18, and 2 meters above the water, respectively. Write the particular equation expressing distance in terms of time.  
 b. How high is the diving board? Justify your answer.  
 c. What is the highest Jack gets above the water?  
 d. When does he hit the water?  
 e. Draw the graph of the quadratic function using a suitable domain.

### CONCEPTS PROBLEM

**Office Building Problem** Suppose that you work for a company that is planning to operate a new office building. You find that the monthly payments the company must make vary directly with the *square* of the number of stories in the building. That is, the general equation is  $\text{payment} = (\text{constant})(\text{stories})^2$ . The amount of money the company will take in each month from people renting office space will vary *linearly* with the number of stories. The *profit* the company will make each month equals the rent payments it takes in minus the monthly payments it pays out.

- a. Write general equations expressing in terms of number of stories  
 i. the monthly payments you pay out,  
 ii. the rent payments you take in, and  
 iii. the monthly profit.  
 b. Tell in words how the monthly profit varies with the number of stories.  
 c. You predict monthly profits of 4, 9, and 12 thousand dollars

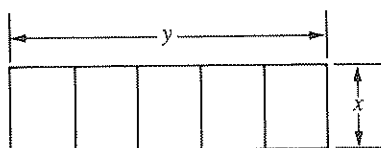
for buildings of 2, 3, and 4 stories, respectively. Find the particular equation expressing monthly profit as a function of the number of stories.

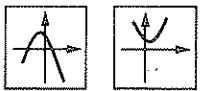
- d. Will the profit ever equal \$15 thousand per month? Justify your answer.
- e. What number of stories gives the *maximum* monthly profit?
- f. Between what two numbers of stories will your monthly profit be positive?
- g. If the building is 7 stories high, how much would you *receive* per month in rent, and how much would you *pay* out per month in monthly payments?
- h. Plot a graph of this function, using a suitable domain.

## CHAPTER TEST

Algebra II is a study of functions. You have been studying quadratic functions.

- T1. Write the general equation for a quadratic function.
- T2. For  $y = 8x^2 - 80x + 207$ , transform to vertex form and write the coordinates of the vertex.
- T3. If  $z = p^2 + 7p + 5$ , find the  $p$ -intercepts.
- T4. Given  $f(x) = 3x^2 - 7x + 11$ ,
  - a. Find  $f(-9)$ .
  - b. Find the values of  $x$  for which  $f(x) = 5$ .
- T5. Suppose that  $y = 7x^2 - 19x + 23$ . Use the discriminant in an appropriate manner to show that there *are* real values of  $x$  for which  $y = 14$ . Are these values *rational* numbers or *irrational* numbers? Justify your answer.
- T6. Given that the vertex of a parabola is at  $(-4, 11)$ , and that the  $y$ -intercept is 3, sketch the parabola.
- T7. Solve  $x^2 + 4x + 104 = 0$  if the domain of  $x$  is the complex numbers.
- T8. **Motel Problem** A small motel is to be built as shown in the sketch with two long walls  $y$  feet long each and 6 short walls  $x$  feet long each. The total length of the walls is to be 300 feet.





- Write an equation expressing  $y$  in terms of  $x$ . What kind of function is this?
- Let  $A(x)$  be the number of square feet area taken up by the motel. Write the particular equation for  $A(x)$ . What kind of function is this?
- Find  $A(10)$  and  $A(30)$ .
- What value of  $x$  makes  $A(x)$  a maximum? What is this maximum area?
- What values of  $x$  make  $A(x)$  equal 0?
- Sketch the graph of function  $A$  in a reasonable domain.

5-9

## CUMULATIVE REVIEW: CHAPTERS 1 THROUGH 5

The following exercise may be considered to be a "final exam" covering the materials on linear and quadratic functions, and the properties leading up to these. If you are thoroughly familiar with the material in Chapters 1 through 5, you should be able to work all of these problems in about 2 to 3 hours.

## EXERCISE 5-9

- Calvin Butterball and Phoebe Small are studying for their algebra examination. In the table on the following page they give the answers indicated to the questions on the left. Tell which, if either, of them is right. Use "Calvin," "Phoebe," "Both," or "Neither."
- Given the statement, "If  $R$  is a relation, then  $R$  is a function:"
  - Draw a graph which shows clearly why the statement is *false*.
  - Write the *converse* of the statement.
  - Is the converse true or false? Explain.
- You have learned that graphs of two linear functions are parallel if their slopes are equal. It is also true that the graphs of two linear functions are *perpendicular* if the slope of one is equal to the *negative* of the *reciprocal* of the other slope. Given the equations

$$y = \frac{2}{3}x - 4 \quad (\text{A}), \quad 3y - 2x = 6 \quad (\text{C}),$$

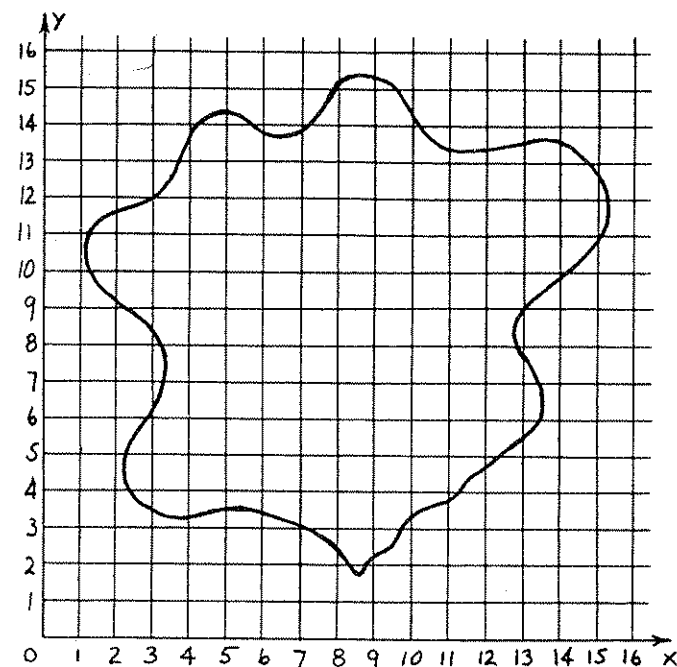
$$2x + 3y = 12 \quad (\text{B}), \quad y - 5 = -\frac{3}{2}(x + 1) \quad (\text{D}).$$

Problem	Calvin	Phoebe
a. $17ax + 34a^2 = 17a(x + 2a)$	distributivity	associativity
b. $(xy + z) + w = xy + (z + w)$	commutativity	associativity
c. $2 + ab = 2 + ba$	commutativity	reflexive property
d. Either $x > 0$ , $x < 0$ , or $x = 0$	comparison	trichotomy
e. $xy \in \{\text{real numbers}\}$	closure	agreement
f. $\frac{xy}{ab} = xy \cdot \frac{1}{ab}$	reciprocal of a product	definition of division
g. $xy + (-xy) = 0$	additive inverses	multiplicative inverses
h. A field axiom	reflexive	symmetric
i. An axiom which is <i>not</i> a field axiom	transitive	closure
j. Name for $4^{\text{th}}$ degree	quadratic	quintic
k. Degree of $3^5x^4y^3 + z^6$	6	12
l. $\sqrt{x^2} =$	$x$	$ x $
m. Set containing $\sqrt{-7}$	{irrational numbers}	{imaginary numbers}
n. If $-2x < 6$ , then	$x > -3$	$x < -3$

- Find the slope of each graph.
  - From the slopes, tell which (if any) of the graphs are parallel, and which (if any) are perpendicular. Use symbols such as  $A \parallel B$ , or  $A \perp B$ .
  - Plot a graph of each equation on the *same* coordinate system.
4. The following sketch shows the feasible region for a problem involving the manufacture of two products. This is similar to a linear programming problem, but the boundaries of the region are not linear. If the profit for this process is given by

$$P = 30x - 50y + 500,$$

- Trace or copy the graph. Then shade the portion of the feasible region in which the profit is more than 300.
- Find the point with integer coordinates that gives the maximum profit, and tell what this profit equals.



5. The following is a proof that a number divided by itself equals 1. Supply a reason for each step. Tell whether or not the reason is an axiom, and if so, whether it is a field axiom.

Prove that

$$\frac{n}{n} = 1.$$

*Proof:*

$$\begin{aligned} \frac{n}{n} &= n \cdot \frac{1}{n} \\ &= 1 \end{aligned}$$

$$\therefore \frac{n}{n} = 1.$$

6. The grade you could make on an algebra final exam is related to the number of days which elapse after the last day of classes if you don't study until you take the exam. Draw a reasonable graph showing how these two variables are related.
7. Assume that your height and your age are related by a *linear* function. Consulting your health records, you find that at age  $A = 5$  your height was  $H = 39$  inches, and when  $A = 9$ ,  $H = 55$  inches.



- a. Write an equation expressing the dependent variable in terms of the independent variable.
  - b. Predict your height at age 16.
  - c. What does the  $H$ -intercept equal, and what does it represent in the real world?
  - d. Since you are using a linear function as a model, what are you assuming about the rate at which you grow?
  - e. What fact in the real world sets an upper bound on the domain in which this linear model gives reasonable answers?
8. For the quadratic function  $y = -2x^2 - 12x - 10$ ,
- a. Transform the equation to the form  $y - k = a(x - h)^2$ .
  - b. Find the vertex.
  - c. Find the  $x$  and  $y$ -intercepts.
  - d. Sketch the graph.
  - e. By using what you have learned about graphing linear inequalities, shade the region on your graph paper corresponding to the solution set of the *quadratic* inequality

$$y \geq -2x^2 - 12x - 10.$$

9. S. Bones is the doctor in Deathly, Ill., a suburb of Chicago. One day, John Garfinkle comes in with a high fever. Dr. Bones takes a blood sample and finds that it contains 1300 flu viruses per cubic millimeter and is *increasing*. John immediately gets a shot of penicillin. The virus count should continue to increase for awhile, then (hopefully!) level off and go back down. After 5 minutes the virus count is up to 1875, and after 5 more minutes, it is 2400. Assume that the virus count varies quadratically with the number of minutes since the shot.
- a. Write an equation expressing the number of viruses per cubic millimeter in terms of the number of minutes since the shot.
  - b. Dr. Bones realizes that if the virus count ever reaches 4500, John must go to the hospital. Must he go? Explain.
  - c. According to your model, when will his flu be completely cured?
  - d. Draw a graph of this quadratic function.
  - e. Tell the range and domain of this quadratic function.
10. For each of the following equations, calculate the discriminant. Then, without actually solving the equation, tell what *kind* of numbers (rational, irrational, imaginary) will be in the solution set.
- a.  $3x^2 - 2x - 8 = 0$
  - b.  $3x^2 - 2x + 8 = 0$
  - c.  $x^2 - 6x + 2 = 0$
  - d.  $x^2 - 6x + 9 = 0$

# 6



## Exponential and Logarithmic Functions

*The polynomial functions you have studied so far have had variables raised to constant powers. In this chapter you will encounter exponential functions that have **constants** raised to **variable** powers. Since variables can be negative numbers or fractions, you must invent meanings for these kinds of exponents. The tool that allows you to operate with such powers is called the "logarithm." The resulting exponential functions are reasonable mathematical models for things from figuring out compound interest to predicting the temperature of a cup of coffee.*

