Unit 5

Quadratic Functions

Unit Overview

In this unit you will study a variety of ways to solve quadratic functions and apply your learning to analyzing real world problems.

Academic Vocabulary

Add these words and others you encounter in this unit to your vocabulary notebook.

- parabola
- parent function
- quadratic formula
- quadratic function
- real roots of an equation
- transformation
- vertex of a parabola

Essential Questions

- How are quadratic functions used to model, analyze and interpret mathematical relationships?
- Why is it advantageous to know a variety of ways to solve and graph quadratic functions?

EMBEDDED ASSESSMENTS

This unit has two embedded assessments, following Activities 5.2 and 5.5. They will allow you to demonstrate your understanding of graphing, identifying, and modeling quadratic functions, and solving quadratic equations.

UNIT 5

Getting Ready

Write your answers on notebook paper. Show your work.

1. Find the products.
 a. $(x - 2)(3x + 5)$
 b. $2y(y + 6)(y - 1)$

2. Factor each polynomial.
 a. $2x^2 + 14x$
 b. $3x^2 - 75$
 c. $x^2 + 7x + 10$

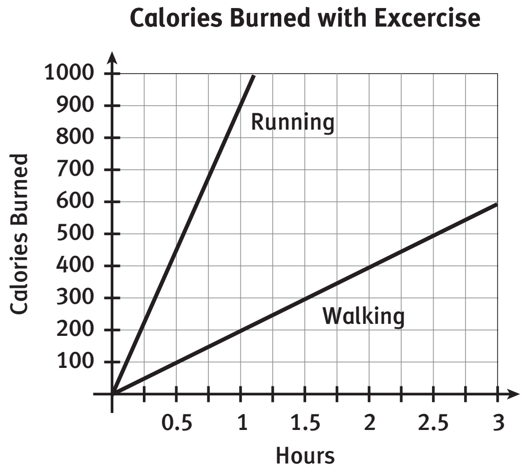
3. If $f(x) = 3x - 5$, find each value.
 a. $f(4)$
 b. $f(-2)$

4. Solve the equation. $4x - 5 = 19$

5. Solve the inequality.
 $\frac{1}{3}x + 9 > 13$

6. Explain how to graph $2x + y = 4$.

The following graph compares calories burned when running and walking at constant rates of 10 mi/h and 2 mi/h, respectively.



7. What does the ordered pair (1.5, 300) represent on this graph?

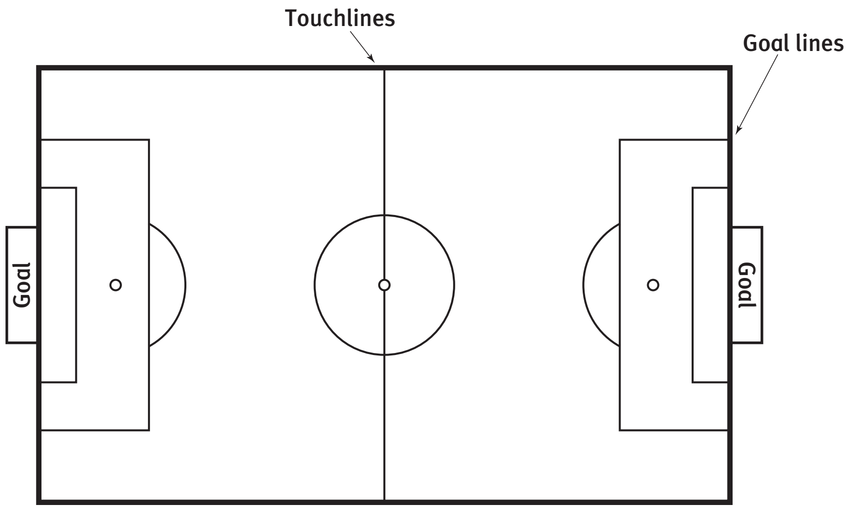
8. How many calories would be burned after four hours when running and walking?

Introduction to Quadratic Functions
Touchlines

SUGGESTED LEARNING STRATEGIES: Self/Peer Revision, Marking the Text, Activating Prior Knowledge

My Notes

Coach Wentworth coaches girls' soccer and teaches algebra. Soccer season is starting, and she needs to mark the touchlines and goal lines for the soccer field. Coach Wentworth can mark 320 yards for the total length of all the touchlines and goal lines. She would like to mark the field with the largest possible area.



FIFA regulations require that all soccer fields be rectangular in shape.

1. How is the perimeter of a rectangle determined? How is the area of a rectangle determined?

CONNECT TO SPORTS

FIFA stands for *Fédération Internationale de Football Association* (International Federation of Association Football) and is the international governing body of soccer.

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Self/Peer Revision, Look for a Pattern

My Notes

2. Complete the table below for rectangles with the given side lengths. The first row has been completed for you.

Length (yards)	Width (yards)	Perimeter (yards)	Area (square yards)
10	150	320	1500
20		320	
40		320	
60		320	
80		320	
100		320	
120		320	
140		320	
150		320	
l		320	

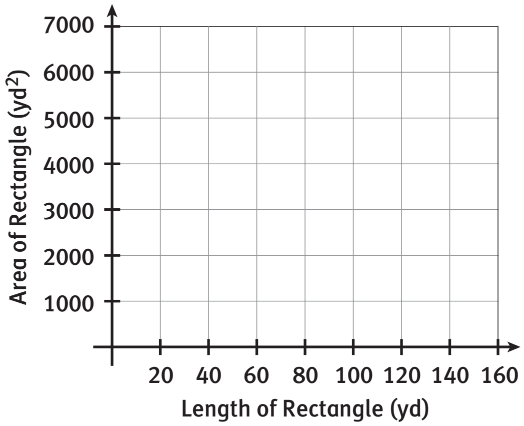
3. Describe any patterns that you notice in the table above.

4. Is a 70-yd by 90-yd rectangle the same as a 90-yd by 70-yd rectangle? Explain your reasoning.

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Self/Peer Revision, Look for a Pattern

My Notes

5. Graph the data from the table in Item 2 as ordered pairs.



6. Are the data in Items 2 and 5 linear? Explain why or why not.

7. Describe any patterns you see in the graph above.

8. What appears to be the largest area from the data in Items 2 and 5?

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Summarize/Paraphrase/Retell

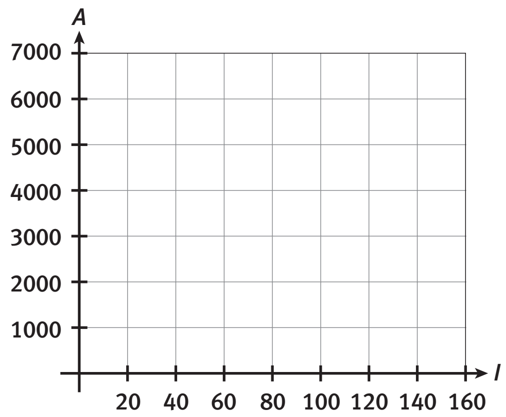
My Notes

9. Write a function $A(l)$ that represents the area of a rectangle whose length is l and whose perimeter is 320.

TECHNOLOGY TIP

Be sure that the RANGE on your calculator's graph matches the ranges shown in the grid at the right.

10. Use a graphing calculator to graph $A(l)$. Sketch the graph on the grid below.



ACADEMIC VOCABULARY

quadratic function

The function $A(l)$ is called a **quadratic function** because it contains a term to the second degree (an x^2 term). The **standard form of a quadratic function** is $y = ax^2 + bx + c$ or $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

11. Write the function $A(l)$ in standard form. What are the values of a, b, and c?

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Guess and Check, Marking the Text, Summarize/Paraphrase/Retell

My Notes

ACADEMIC VOCABULARY

parabola
vertex of a parabola

The graph of a quadratic function is a curve called a **parabola**. A parabola has a point at which a maximum or minimum value of the function occurs. That point is called the **vertex of a parabola**. The y-value of the vertex is the **maximum** or **minimum** of the function.

12. What is the vertex of the graph of $A(l)$ in Item 10? Does the vertex represent a maximum or a minimum of the function?

FIFA regulations state that the length of the touchline of a soccer field must be greater than the length of the goal line.

13. Can Coach Wentworth use the rectangle that represents the largest area of $A(l)$ for her soccer field? Why or why not?

FIFA regulations also state that the length of the touchlines of a soccer field must be at least 100 yd but no more than 130 yd. The goal lines must be at least 50 yd but no more than 100 yd.

14. Find the dimensions of the FIFA regulation soccer field with largest area. Support your reasoning with multiple representations.

CHECK YOUR UNDERSTANDING

Use notebook paper or grid paper to write your answers. Show your work.

1. State whether the data in each table are linear. Explain why or why not.

a.

x	y
0	5
1	2
2	-1
3	-4
4	-7

b.

x	y
0	5
1	2
2	1
3	2
4	5

2. Graph the data in the table in Item 1(b) above. What is the least value of the data on the graph?

3. Identify whether each function is quadratic.

a. $f(x) = 6x + 12$

b. $g(x) = 6x^2 + 12$

c. $h(x) = \frac{6}{x^2} + 12$

4. Write each quadratic function in standard form.

a. $f(x) = 5 - 2x + x^2$

b. $g(x) = 3x^2 + 8 - 9x$

c. $h(x) = 4x + 7 - 2x^2$

d. $l(x) = \frac{2x + 5x^2}{2}$

5. Complete the tables. Then graph the quadratic functions.

a. $f(x) = x^2 + 2x + 3$

x	$f(x)$
-3	
-2	
-1	
0	
1	

b. $f(x) = -x^2 - 4x - 3$

x	$f(x)$
-4	
-3	
-2	
-1	
0	

6. Identify the maximum or minimum values of the quadratic functions in Item 5.

7. **MATHEMATICAL REFLECTION** What characteristics of the graph of a quadratic function distinguish it from that of a linear function?

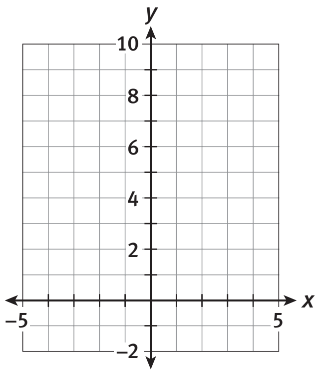
Graphing $y = ax^2 + c$
Transformers

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Quickwrite, Self/Peer Revision

My Notes

1. Complete the table for $y = x^2$. Graph the function. The equation and the graph are referred to as the **parent** quadratic function.

x	y
−3	
−2	
−1	
0	
1	
2	
3	



ACADEMIC VOCABULARY

A **parent function** is the most basic function of a particular category or type. For example, the parent linear function is $y = x$.

2. Using mathematical vocabulary, describe the graph of $y = x^2$.

3. Complete the second and third columns in the table for $y = x^2$. Use your results to explain why the function is not linear.

x	y	Difference between consecutive y values ("First Differences")	
−3		-------------	
−2			
−1			
0			
1			
2			
3			

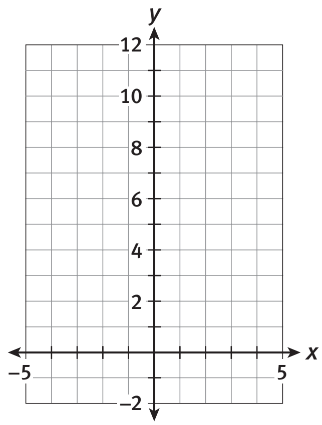
SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Quickwrite, Self/Peer Revision

My Notes

4. Label the fourth column in the table in Item 3 as "Second Differences." Complete the table by finding the change in consecutive values in the third column. What do you notice about the values?

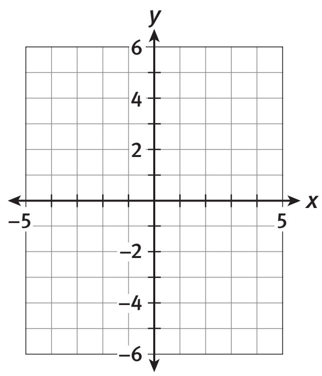
5. Complete the table for $y = x^2$ and $y = x^2 + 3$. Then graph each function on the same coordinate grid.

x	x^2	$x^2 + 3$
−3		
−2		
−1		
0		
1		
2		
3		



6. Complete the table for $y = x^2$ and $y = x^2 - 4$. Then graph each function on the same coordinate grid.

x	x^2	$x^2 - 4$
−3		
−2		
−1		
0		
1		
2		
3		



SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Think/Pair/Share, Predict and Confirm, Create Representations

My Notes

7. Compare the functions in Items 5 and 6 to the parent function $y = x^2$. Describe any patterns you notice in the following.

a. equations

b. tables

c. graphs

The changes to the parent function in Items 5 and 6 are examples of vertical **translations**.

MATH TERMS

A **translation** of a graph is a change that shifts the graph horizontally, vertically, or both. A translation preserves the shape of the graph.

8. How does the value of c in the equation $y = x^2 + c$ change the graph of the parent function $y = x^2$?

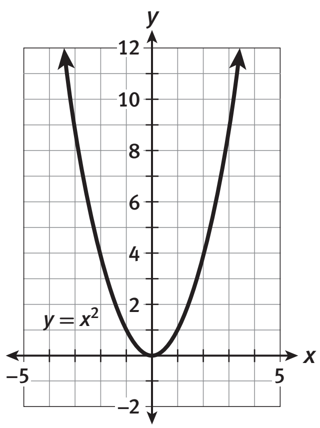
TRY THESE A

Predict the translations of the graph of $y = x^2$ for each equation. Then confirm your predictions by graphing each equation.

a. $y = x^2 - 2$ **b.** $y = x^2 + 4$

9. Complete the table for $y = x^2$ and $y = 2x^2$. Then graph $y = 2x^2$ on the coordinate grid below ($y = x^2$ is already graphed for you).

x	x^2	$2x^2$
−2		
−1		
0		
1		
2		

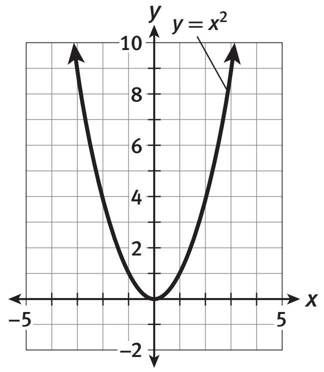


SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Quickwrite, Self/Peer Revision

My Notes

10. Complete the table for $y = x^2$ and $y = \frac{1}{2}x^2$. Then graph $y = \frac{1}{2}x^2$ on the coordinate grid below ($y = x^2$ is already graphed for you).

x	x^2	$\frac{1}{2}x^2$
−2		
−1		
0		
1		
2		



11. Compare the functions in Items 9 and 10 to the parent function $y = x^2$. Describe any patterns you notice in the following.

a. equations

b. tables

c. graphs

The change to the parent function in Item 9 is a **vertical stretch** by a given factor and the change in Item 10 is a **vertical shrink** by a given factor.

MATH TERMS
A **vertical shrink** or **stretch** will change the shape of the graph. The new graph will not be congruent to the graph of the parent function.

12. How does the value of a in the equation $y = ax^2$ change the graph of the parent function $y = x^2$?

TRY THESE **B**

Identify the change in the graph of $y = x^2$ for each equation. Then graph each to verify.

a. $y = 4x^2$

b. $y = \frac{1}{4}x^2$

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Look for a Pattern, Quickwrite, Self/Peer Revision

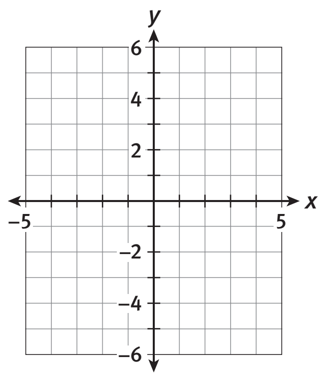
My Notes

ACADEMIC VOCABULARY

transformation

13. A change in the position, size or shape of a parent graph is a **transformation**. Identify the transformations that have been introduced so far in this activity.

14. Graph $y = -x^2$ on the coordinate grid.



15. Compare the quadratic function in Item 14 and its graph to the parent function $y = x^2$.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Self/Peer Revision, Predict and Confirm, Create Representations

My Notes

MATH TERMS

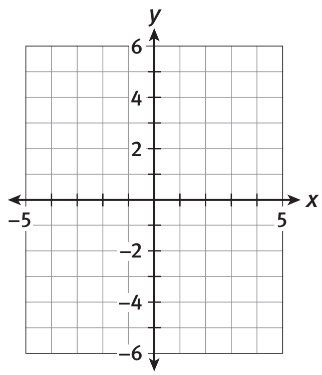
A **reflection** of a graph is the mirror image of the graph over a line. A reflection preserves the shape of the graph.

The change to the parent function in Item 14 is a **reflection** over the x-axis.

16. How does the sign of a in the equation $y = ax^2$ affect the graph?

17. How would you transform the graph of $y = x^2$ to produce the graph of $y = 2x^2 - 5$?

18. Use the transformations you described in Item 17 to graph the function $y = 2x^2 - 5$.

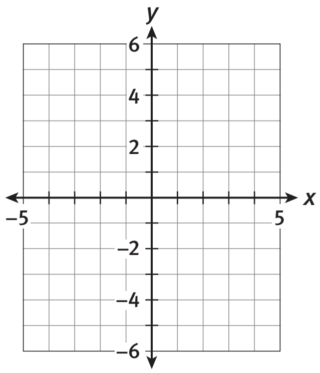


SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Self/Peer Revision, Create Representations

My Notes

19. How would you transform of the graph of to $y = x^2$ produce the graph of the function $y = -\frac{1}{2}x^2 + 4$?

20. Use the transformations you described in Item 19 to graph the function $y = -\frac{1}{2}x^2 + 4$.



CHECK YOUR UNDERSTANDING

Use notebook paper or grid paper to write your answers. Show your work.

To obtain each function, identify the transformations that need to be applied to the graph of the parent function $y = x^2$. Then graph each function.

1. $y = 5x^2$

2. $y = \frac{1}{3}x^2$

3. $y = x^2 + 10$

4. $y = x^2 - 8$

5. $y = 2x^2 - 1$

6. $y = \frac{1}{2}x^2 + 3$

7. $y = -x^2 + 4$

8. $y = -3x^2 + 5$

Write the equation of the function described by each of the following transformations of the graph of $y = x^2$.

9. Translated up 9 units.

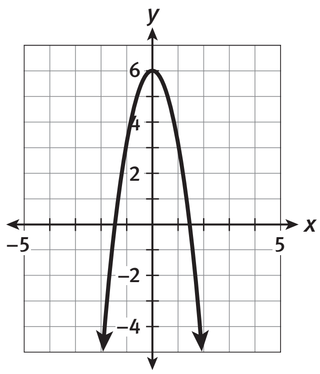
10. Shrunk vertically by a factor of $\frac{1}{3}$.

11. Stretched vertically by a factor of 5 and translated down 12 units.

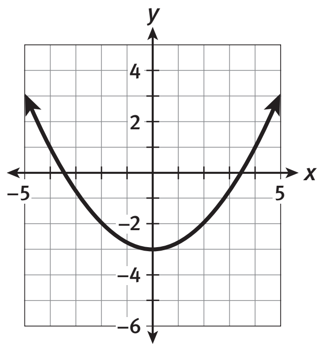
12. Reflected over x-axis, shrunk vertically by a factor of $\frac{3}{4}$ and translated up π units.

Write an equation for each function displayed.

13.



14.



15. **MATHEMATICAL REFLECTION** Identify and compare the transformations introduced in this activity.

Embedded Assessment 1

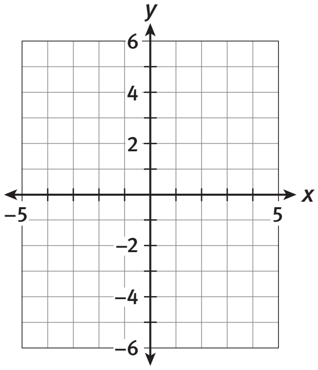
Use after Activity 5.2.

Graphing Quadratics

QUADRATIC INTRODUCTION

1. Given the function $y = -x^2 + 4$.

a. Graph the function and explain how you graphed it.



b. Identify the vertex.

c. Write the maximum or minimum of the function.

2. Identify the table that represents a quadratic function and explain your choice.

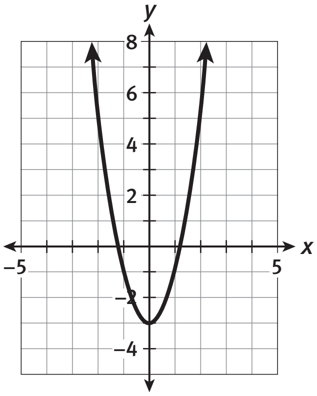
a.

x	*y*
−2	−1
−1	−3
0	−5
1	−3
2	−1

b.

x	*y*
−2	3
−1	−3
0	−5
1	−3
2	3

3. The graph below is for a function of the form $y = ax^2 + c$. Find a and c and write the equation of the function.



Embedded Assessment 1

Use after Activity 5.2.

Graphing Quadratics

QUADRATIC INTRODUCTION

	Exemplary	Proficient	Emerging
Math Knowledge #1b, c, 3	The student: • Correctly identifies the vertex, maximum, and minimum of the function, based on the graph. (1b, c) • Writes the correct function. (3)	The student: • Correctly identifies only two of the vertex, maximum, and minimum, based on the graph. • Writes a quadratic function, based on incorrect *a* or *c*.	The student: • Correctly identifies only one of the vertex, maximum, or minimum, based on the graph. • Writes a quadratic function, based on incorrect *a* and *c*.
Problem Solving #2, 3	The student: • Correctly identifies the quadratic function. (2) • Determines the correct values for *a* and *c*. (3)	The student determines the correct value for *a* or *c*.	The student: • Does not identify the correct function. • Determines the correct value for neither *a* nor *c*.
Representations #1a	The student graphs the function correctly. (1a)	The student graphs part of the function correctly.	The student's graph is completely incorrect.
Communication #1a, 2	The student: • Explains clearly the method used to graph the function. (1a) • Explains clearly the method used to identify the quadratic function. (2)	The student attempts both explanations, but only one is correct and complete.	The student attempts both explanations, but neither is correct and complete.

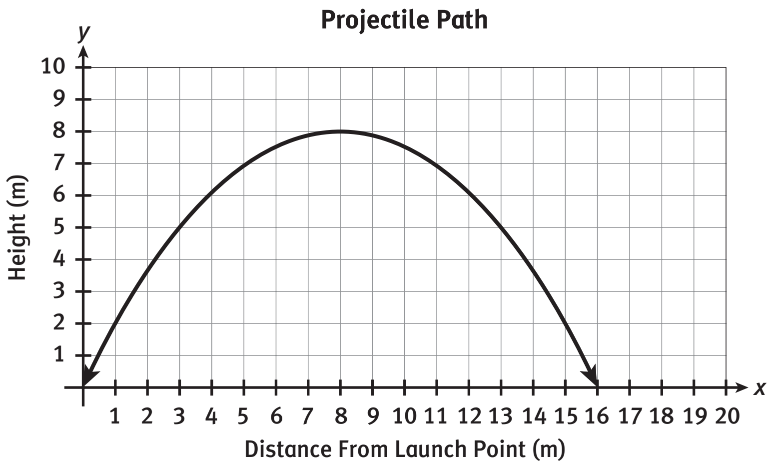
Solving Quadratic Equations
Trebuchet Trials

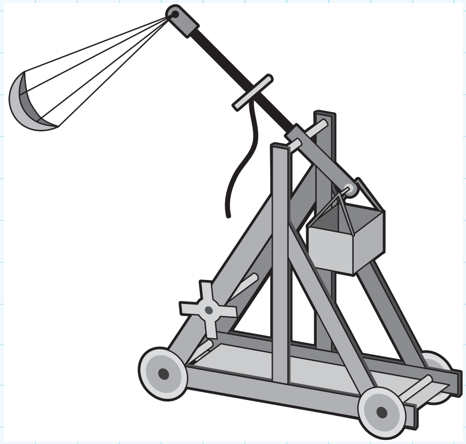
SUGGESTED LEARNING STRATEGIES: Shared Reading, Visualize, Summarize/Paraphrase/Retell, Think/Pair/Share, KWL Chart

My Notes

Carter, Alisha, and Joseph are building a *trebuchet* for an engineering competition. A trebuchet is a medieval siege weapon that uses gravity to launch an object through the air. When the counterweight at one end of the throwing arm drops, the other end rises and a projectile is launched through the air. The path the projectile takes through the air is modeled by a parabola.

To win the competition, the team must build their trebuchet according to the competition specifications to launch a small projectile as far as possible. After conducting experiments that varied the projectile's mass and launch angle, the team discovered that the ball they were launching followed the path given by the quadratic equation $y = -\frac{1}{8}x^2 + 2x$.





CONNECT TO PHYSICS

The throwing arm of a trebuchet is an example of a class I lever.

1. How far does the ball land from the launching point?

2. What is the maximum height of the ball?

3. What are the x-coordinates of the points where the ball is on the ground?

SUGGESTED LEARNING STRATEGIES: Quickwrite, Note Taking, Group Presentation, Debriefing

My Notes

To determine how far the ball lands from the launching point, you can solve the equation $-\frac{1}{8}x^2 + 2x = 0$, because the height y equals 0.

4. Verify that $x = 0$ and $x = 16$ are solutions to this equation.

5. Without the graph, could you determine these solutions? Explain.

MATH TERMS
A polynomial is in **factored form** when it is expressed as the product of one or more polynomials.

MATH TERMS
Zero Product Property

The **factored form** of a polynomial equation provides an effective way to determine the values of x that make the equation equal 0.

Zero Product Property
If $ab = 0$, then either $a = 0$ or $b = 0$.

You can use this property to solve equations in factored form.

EXAMPLE 1

Solve $-\frac{1}{8}x^2 + 2x = 0$ by factoring.

Step 1: *Factor.* $-\frac{1}{8}x^2 + 2x = x(-\frac{1}{8}x + 2) = 0$

Step 2: *Apply the Zero Product Property.* $x = 0$ or $-\frac{1}{8}x + 2 = 0$

Step 3: *Solve each equation for x.* $x = 0$ or $-\frac{1}{8}x + 2 = 0$

$-\frac{1}{8}x = -2$

$x = (-2)(-8) = 16$

Solution: $x = 0$ or $x = 16$

TRY THESE A

Solve each quadratic equation by factoring.

a. $x^2 - 5x - 14 = 0$

b. $3x^2 - 6x = 0$

c. $x^2 + 3x = 18$

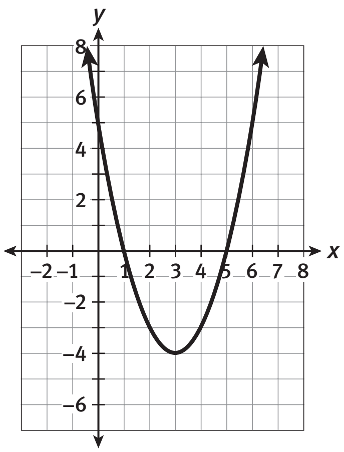
SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Think/Pair/Share, Quickwrite, Group Presentation

My Notes

6. How do the solutions to the projectile path equation $-\frac{1}{8}x^2 + 2x = 0$ in Example 1 relate to the equation's graph?

The graph of the function $y = x^2 - 6x + 5$ is shown below.

7. What are the x-intercepts of the graph?



8. What is the x-coordinate of the vertex?

9. Describe the location of x-coordinate of the vertex with respect to the two x-intercepts.

10. Solve the related quadratic equation $x^2 - 6x + 5 = 0$ by factoring.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Vocabulary Organizer, Note Taking, Quickwrite, KWL Chart

My Notes

11. How do the solutions you found in Item 10 relate to the x-intercepts of the graph next to Item 7?

ACADEMIC VOCABULARY

real roots of an equation

The x-intercepts of a quadratic function $y = ax^2 + bx + c$ are the **zeros of the function**. The solutions of a quadratic equation $ax^2 + bx + c = 0$ are the **real roots of the equation**.

12. The quadratic function $y = ax^2 + bx + c$ is related to the equation $ax^2 + bx + c = 0$ by letting y equal zero.

a. Why do you think the x-intercepts are called zeros?

b. Complete the statement: The ______________ of an equation are equal to the ______________ of the function.

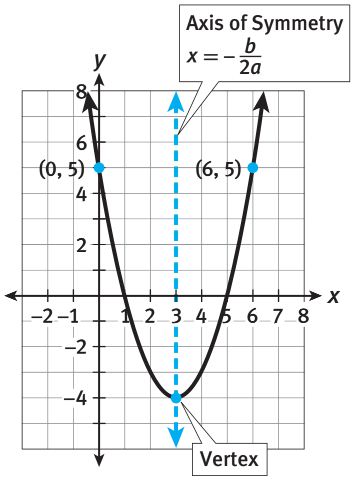
MATH TERMS

axis of symmetry

The **axis of symmetry** of a parabola of the function $y = ax^2 + bx + c$ is the vertical line that passes through the vertex.

The equation for the axis of symmetry is $x = -\frac{b}{2a}$.

The vertex is on the axis of symmetry. Therefore, the x-coordinate of the vertex is $-\frac{b}{2a}$.



Each point on a quadratic graph will have a mirror image point with the same y-coordinate that is equidistant from the axis of symmetry. You can see that (0, 5) is reflected over the axis of symmetry to get the point (6, 5) on the graph.

MATH TIP

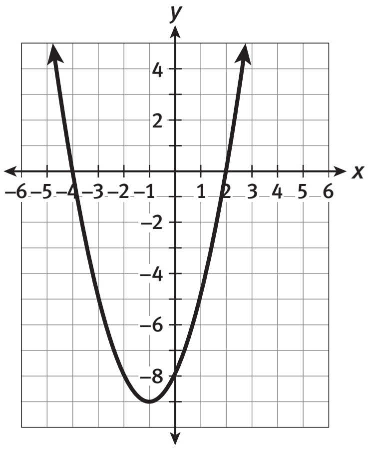
Substitute the x-coordinate of the vertex into the quadratic equation to find the y-coordinate of the vertex.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think/Pair/Share, Guess and Check

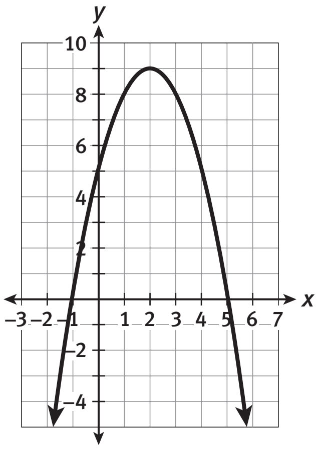
My Notes

13. For each function, identify the zeros graphically. Confirm your answer by setting the function equal to 0 and solving by factoring.

a. $y = x^2 + 2x - 8$



b. $y = -x^2 + 4x + 5$



14. For each graph in Item 13, determine the x-coordinate of the vertex by finding the x-coordinate exactly in the middle of the two zeros. Confirm your answer by calculating the value of $-\frac{b}{2a}$.

a. $y = x^2 + 2x - 8$

b. $y = -x^2 + 4x + 5$

15. For each graph in Item 13, determine the y-coordinate of the vertex from the graph. Confirm your answer by evaluating the function at $x = -\frac{b}{2a}$.

a. $y = x^2 + 2x - 8$

b. $y = -x^2 + 4x + 5$

SUGGESTED LEARNING STRATEGIES: Note Taking

My Notes

If a quadratic function can be written in factored form, you can graph it by finding the vertex and the zeros.

EXAMPLE 2

Graph the quadratic function $y = x^2 - x - 12$.

Step 1: *Find the axis of symmetry using* $x = -\frac{b}{2a}$.

The axis of symmetry is $x = 0.5$.

$y = x^2 - x - 12$

$a = 1, b = -1$

$x = -\frac{(-1)}{2(1)} = \frac{1}{2} = 0.5$

Step 2: *Find the vertex.*

The x-coordinate is 0.5. Substitute 0.5 for x to find the y-coordinate.

$y = (0.5)^2 - (0.5) - 12$

$= 0.25 - 0.5 - 12$

$= -12.25$

The vertex is (0.5, −12.25).

Step 3: *Find the zeros of the function.*

Set the function equal to 0 and solve.

The zeros are −3 and −4.

$x^2 - x - 12 = 0$

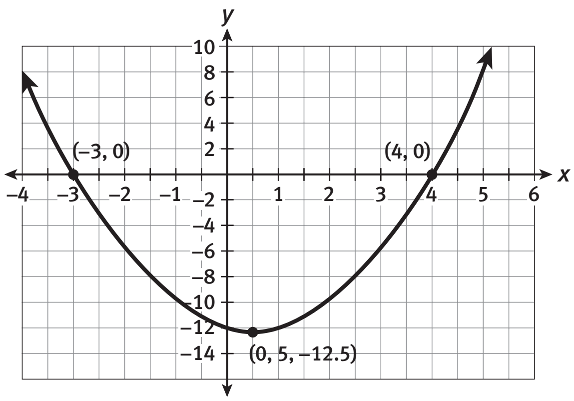
$(x - 4)(x + 3) = 0$

$x - 4 = 0$ or $x + 3 = 0$

$x = 4$ or $x = -3$

Step 4: *Graph the function.*

Plot the vertex (0.5, −12.25) and the points where the function crosses the x-axis (−3, 0) and (4, 0). Connect the points with a smooth parabolic curve.



SUGGESTED LEARNING STRATEGIES: Create Representations, Group Presentation, Shared Reading, Identify a Subtask

My Notes

TRY THESE **B**

a. Check the Example 2 graph by plotting two more points on the graph. First choose an x-coordinate of a point, and then find the y-coordinate by evaluating the equation. Plot the reflection of the point over the axis of symmetry to get another point. Verify that these points are on the graph.

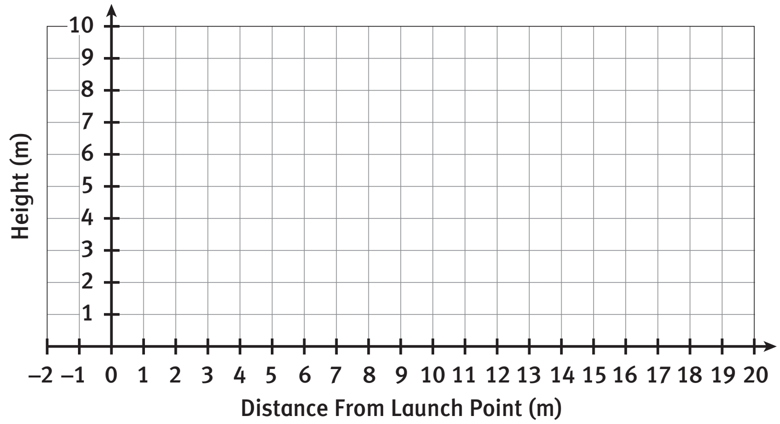
Graph the quadratic functions by finding the vertex and the zeros. Check your graphs.

b. $y = x^2 - 2x - 8$ **c.** $y = 4x - x^2$ **d.** $y = x^2 + 4x - 5$

Joseph, Carter and Alisha tested a new trebuchet designed to launch the projectile even further. They also refined their model to reflect a more accurate launch height of 1 m. The new projectile path is given by the function $y = -\frac{1}{19}(x^2 - 18x - 19)$.

16. Graph the projectile path on the coordinate axes below.

Distance Projectile Path



SUGGESTED LEARNING STRATEGIES: Summarize/Paraphrase/Retell, Quickwrite

My Notes

17. Last year's winning trebuchet launched a projectile a horizontal distance of 19.5 m. How does the team's trebuchet compare to last year's winner?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

Solve each equation by factoring.

1. $x^2 - 11x + 30 = 0$
2. $x^2 - 9x - 10 = 0$
3. $2x^2 - 4x = 0$
4. $x^2 - 7x = 18$
5. $5x = 24 - x^2$

What are the *x*-intercepts of each quadratic function?

6. $y = x^2 - 49$
7. $y = x^2 - 4x + 3$
8. $y = x^2 - 5x$

Graph each function.What is the axis of symmetry and vertex of each quadratic function?

9. $y = x^2 - 10x$
10. $y = x^2 - 4x - 32$

Graph each function. What is the axis of symmetry and vertex of each quadratic function?

11. $y = x^2 + x - 12$
12. $y = 6x - x^2$
13. MATHEMATICAL REFLECTION How do the zeros of a quadratic equation relate to the graph of the related quadratic function?

Solving Quadratic Equations
Keeping It Quadratic

ACTIVITY
5.4

SUGGESTED LEARNING STRATEGIES: Guess and Check, Group Presentation, Quickwrite

My Notes

1. Solve each equation. Be prepared to discuss your solution methods with your classmates.

a. $x^2 = 49$

b. $x^2 = 100$

c. $x^2 = 15$

d. $2x^2 = 18$

e. $x^2 - 4 = 0$

f. $x^2 + 2 = 0$

g. $x^2 + 3 = 3$

MATH TIP

Recall that a solution of an equation makes the equation true. For example, $x + 5 = 7$ has the solution $x = 2$ because $2 + 5 = 7$.

2. Refer to the equations in Item 1 and their solutions.

a. What do the equations have in common?

b. What types of numbers are represented by the solutions of these equations?

c. How many solutions do these types of equations have?

My Notes

SUGGESTED LEARNING STRATEGIES: Note Taking, Graphic Organizer, Simplify the Problem, Marking the Text, Questioning the Text

To solve a quadratic equation of the form $ax^2 + c = 0$, isolate the x^2-term and then take the square root of both sides.

EXAMPLE 1

Solve $3x^2 - 6 = 0$ using the square root method.

Step 1: *Add 6 to both sides.* $3x^2 - 6 = 0$

$3x^2 = 6$

Step 2: *Divide both sides by 3.* $\frac{3x^2}{3} = \frac{6}{3}$

$x^2 = 2$

Step 3: *Take the square root of both sides.* $\sqrt{x^2} = \pm\sqrt{2}$

$x = +\sqrt{2}$ or $-\sqrt{2}$

Solution: $x = +\sqrt{2}$ or $x = -\sqrt{2}$

TRY THESE A

Solve each equation using the square root method.

a. $x^2 - 10 = 1$ **b.** $\frac{x^2}{4} = 1$ **c.** $4x^2 - 6 = 14$

MATH TIP

Every positive number has 2 square roots, the principal square root and its opposite. For example, $\sqrt{5}$ is the principal square root and $-\sqrt{5}$ is its opposite.

When you solve quadratic equations, use the $\pm$ symbol to represent both square roots.

READING MATH

The $\pm$ symbol is read "plus or minus."

5. Quadratic equations can have 0, 1 or 2 real solutions. Fill in the table below with equations from the first page that represent the possible types of solutions.

Number of Solutions	Result when x^2 is isolated	Example(s)
Two	x^2 = positive number	
One	$x^2 = 0$	
No real solutions	x^2 = negative number	

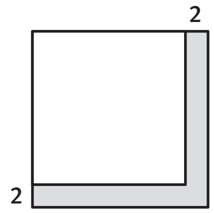
MATH TIP

The square root of a negative number is not a real number, so equations of the form $x^2 = c$ where $c < 0$, have no real solutions.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Note Taking, Quickwrite

My Notes

6. A square frame has a 2-in. border along two sides as shown in the diagram. The total area is 66 in.^{2}. The questions will help you write an equation to find the area of the unshaded square.



a. Label the sides of the unshaded square x.

b. Write an equation for the total area in terms of x.

Area in terms of x	=	Area in square in.

You can solve quadratic equations like the one you wrote in Item 6 by isolating the variable.

EXAMPLE 2

Solve $(x + 2)^2 = 66$ using square roots. Approximate the solutions to the nearest hundredth.

Step 1: *Take the square root of both sides.*

$$(x + 2)^2 = 66$$
$$\sqrt{(x + 2)^2} = \pm\sqrt{66}$$

Step 2: *Subtract 2 from both sides.*

$$x + 2 = \pm\sqrt{66}$$
$$x = -2 \pm \sqrt{66}$$
$$x = -2 + \sqrt{66} \text{ or } x = -2 - \sqrt{66}$$

Step 3: *Use a calculator to approximate the solutions.*

Solution: $x \approx -10.12$ or 6.12

7. Are both solutions to this equation valid in the context of Item 6? Explain.

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Identify a Subtask, Look for a Pattern, Marking the Text, Activating Prior Knowledge, Graphic Organizer

My Notes

TRY THESE B

Solve each equation using square roots.

a. $(x - 5)^2 = 121$

b. $(2x - 1)^2 = 6$

c. $x^2 - 12x + 36 = 2$

As shown in Example 2, quadratic equations are more easily solved with square roots when the side with the variable is a perfect square.

When a quadratic equation is written in the form $x^2 + bx + c = 0$, you can complete the square to transform the equation into one that can be solved using square roots. **Completing the square** is the process of adding a term to the variable side of a quadratic equation to transform it into a perfect square trinomial.

EXAMPLE 3

Solve $x^2 + 10x - 6 = 0$ by completing the square.

Step 1: *Isolate the variable terms.* $x^2 + 10x - 6 = 0$

Add 6 to both sides. $x^2 + 10x = 6$

Step 2: *Transform the left side into a perfect square trinomial.*

Divide the coefficient of the x-term by 2. $10 \div 2 = 5$

Square the 5 to determine the constant. $5^2 = 25$

Complete the square by adding 25 to both sides of the equation. $x^2 + 10x + \square = 6 + \square$

$x^2 + 10x + \boxed{25} = 6 + \boxed{25}$

Step 3: *Solve the equation.* $x^2 + 10x + 25 = 31$

Write the trinomial side in factored form. $(x + 5)(x + 5) = 31$

Write the left side as a square of binomials. $(x + 5)^2 = 31$

Take the square root of both sides. $\sqrt{(x + 5)^2} = \pm\sqrt{31}$

Solve for x. $(x + 5) = \pm\sqrt{31}$

Leave the solutions in $\pm$ form. $x = -5 \pm \sqrt{31}$

Solution: $x = -5 \pm \sqrt{31}$

MATH TIP

Use a graphic organizer to help you complete the square.

	x	5
x	x^2	$5x$
5	$5x$	25

$x^2 + 5x + 5x + 25$
$= (x + 5)(x + 5)$

$x^2 + 10x + 25 = (x + 5)^2$

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Group Presentation, Identify a Subtask, Look for a Pattern

My Notes

TRY THESE **C**

Solve each quadratic equation by completing the square.

a. $x^2 - 8x + 3 = 11$ **b.** $x^2 + 7 = 2x + 8$

Generalizing a solution method into a formula provides an efficient way to perform complicated procedures. You can complete the square on the general form of a quadratic equation $ax^2 + bx + c = 0$ to find a formula for solving all quadratic equations.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x + \frac{b}{a}x + \square = -\frac{c}{a} + \square$$

$$x + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

When $a \neq 0$, the solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

ACADEMIC VOCABULARY

quadratic formula

SUGGESTED LEARNING STRATEGIES: Note Taking, Identify a Subtask, Simplify the Problem

My Notes

To find the solutions of a quadratic equation written in standard form, first identify the values of *a*, *b*, and *c* in the equation and then substitute these values into the quadratic formula.

EXAMPLE 4

Solve $2x^2 - 3x - 5 = 0$ using the quadratic formula.

Step 1: *Identify a, b, and c.* $2x^2 - 3x - 5 = 0$

$a = 2, b = -3, c = -5$

Step 2: *Substitute these values into the Quadratic Formula.* $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)}$

Step 3: *Simplify using the Order of Operations.* $x = \frac{3 \pm \sqrt{9 + 40}}{4}$

$x = \frac{3 \pm \sqrt{49}}{4}$

Step 4: *Write as two expressions and simplify.* $x = \frac{3 + 7}{4}$ or $\frac{3 - 7}{4}$

Solution: $x = \frac{5}{2}$ or -1

Check your answers in the original equation.

$$2\left(\frac{5}{2}\right)^2 - 3\left(\frac{5}{2}\right) - 5 \stackrel{?}{=} 0$$
$$2\left(\frac{25}{4}\right) - 3\left(\frac{5}{2}\right) - 5 \stackrel{?}{=} 0$$
$$\frac{25}{2} - \frac{15}{2} - 5 \stackrel{?}{=} 0$$
$$\frac{10}{2} - 5 \stackrel{?}{=} 0$$
$$0 = 0$$

$$2(-1)^2 - 3(-1) - 5 \stackrel{?}{=} 0$$
$$2(1) - 3(-1) - 5 \stackrel{?}{=} 0$$
$$2 + 3 - 5 \stackrel{?}{=} 0$$
$$0 = 0$$

TRY THESE **D**

Solve using the quadratic formula. Check.

a. $x^2 - 11x - 42 = 0$

b. $2x^2 - 3x - 6 = 0$

SUGGESTED LEARNING STRATEGIES: Note Taking, Group Presentation

My Notes

To apply the quadratic formula, make sure the equation is in standard form $ax^2 + bx + c = 0$. If the expression under the radical sign is not a perfect square, you can write the solutions in simplest form or you can use a calculator to approximate the solutions.

EXAMPLE 5

Solve $x^2 + 3 = 6x$ using the quadratic formula.

Step 1: *Write the equation in standard form.* $x^2 + 3 = 6x$
$x^2 - 6x + 3 = 0$

Step 2: *Identify a, b, and c.* $a = 1, b = -6, c = 3$

Step 3: *Substitute these values into the quadratic formula.* $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}$

Step 4: *Simplify using the order of operations.* $x = \dfrac{6 \pm \sqrt{36 - 12}}{2} = \dfrac{6 \pm \sqrt{24}}{2}$

Step 5: *Write as two expressions.* $x = \dfrac{6 + \sqrt{24}}{2}$ or $\dfrac{6 - \sqrt{24}}{2}$

Solution: Use a calculator to approximate the two solutions.
$x \approx 5.45$ or $x \approx 0.55$

If you do not have a calculator, you can write your solution in simplest form. To write your solution in simplest form, simplify the radicand and then divide out any common factors.

$$x = \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm \sqrt{4 \cdot 6}}{2} = \frac{6 \pm 2\sqrt{6}}{2} = \frac{2(3 \pm \sqrt{6})}{2} = 3 \pm \sqrt{6}$$

TRY THESE E

Solve using the quadratic formula.

a. $3x^2 = 4x + 3$

b. $x^2 + 4x = -2$

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Quickwrite

My Notes

You have learned several methods for solving a quadratic equation. They include factoring, using square roots, completing the square and using the quadratic formula.

8. Solve each equation below using a different method.

a. $x^2 + 5x - 24 = 0$

b. $x^2 - 6x + 2 = 0$

c. $2x^2 + 3x - 5 = 0$

d. $x^2 - 100 = 0$

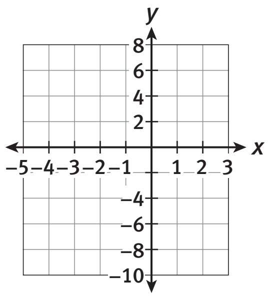
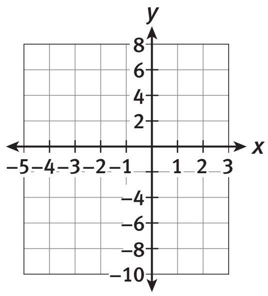
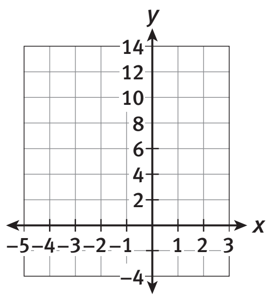
9. How did you decide which method to use on each equation in Item 8?

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Graphic Organizer, Create Representations, Identify a Subtask

My Notes

The expression $\sqrt{b^2 - 4ac}$ in the Quadratic Formula helps you understand the nature of the quadratic equation. The **discriminant**, $b^2 - 4ac$, of a quadratic equation gives information about the number of real solutions, as well as the number of x-intercepts of the related quadratic function.

10. For each equation, solve the equation using one of the solution methods you learned and then complete the rest of the table.

Equation and Solutions	Discriminant	Number of Real Solutions	Graph of Related Quadratic Function
$x^2 + 2x - 8 = 0$			
$x^2 + 2x + 1 = 0$			
$x^2 + 2x + 5 = 0$			

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Group Presentation

My Notes

11. Complete each statement below using the information from the table in Item 10.

If $b^2 - 4ac > 0$, there are _______ real solutions and _______ x-intercepts.

If $b^2 - 4ac = 0$, there are _______ real solutions and _______ x-intercepts.

If $b^2 - 4ac < 0$, there are _______ real solutions and _______ x-intercepts.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper. Show your work.

Solve using the square root method.

1. $x^2 - 5 = 6$

2. $3x^2 - 7 = 0$

3. $2(x - 9)^2 = 100$

Solve by completing the square.

4. $x^2 + 12x - 8 = 0$

5. $x^2 + 4x = 12$

6. $x^2 - 5x + 1 = 8$

Solve using the quadratic formula. State if there are no real solutions.

7. $x^2 + 5x - 3 = 0$

8. $x^2 - 7 = 2x$

9. $2x^2 - 5x + 6 = 0$

10. The width of a rectangle is 5 more than the length. If the area is 20 square units, what are the dimensions of the rectangle?

11. MATHEMATICAL REFLECTION How do you identify which method is best for solving a quadratic equation?

Applying Quadratic Equations
Rockets in Flight

ACTIVITY
5.5

SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Questioning the Text, Look for a Pattern, Quickwrite

My Notes

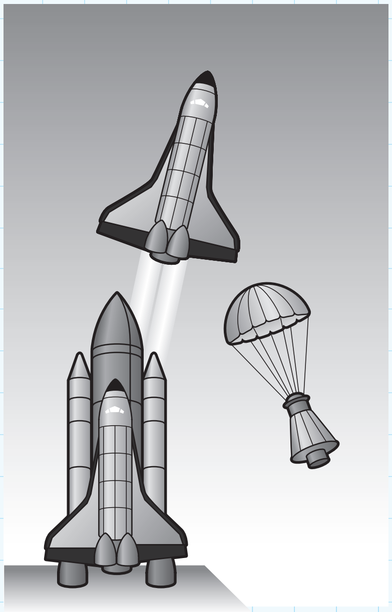
Homer H. Hickam Jr. is a coal miner's son, who lived in West Virginia during the 1950s. After the Russians launched the *Sputnik* satellite, Homer was inspired to learn about model rocketry. After many tries, Homer and his friends discovered how to launch and control the flight path of their model rockets. Homer went on to college and then worked for NASA.

Cooper is a model rocket fan. Cooper's model rockets have single engines and, when launched, can rise as high as 1000 ft depending upon the engine size. After the engine is ignited, it will burn for 3–5 seconds and the rocket will accelerate upward. Once the engine burns out, the rocket will be in *free fall*, because the only acceleration is due to gravity. The rocket has a parachute that will open as the rocket begins to fall back to Earth.

Cooper wanted to track one of his rockets, the *Eagle*, so that he could investigate its time and height while in flight. He installed a device into the nose of the *Eagle* to measure the time and height of the rocket as it fell back to Earth. The device started measuring when the parachute opened. The data for one flight of the *Eagle* is shown in the table below.

CONNECT TO SCIENCE

NASA is the National Aeronautics and Space Administration and is responsible for all space exploration.



The *Eagle*

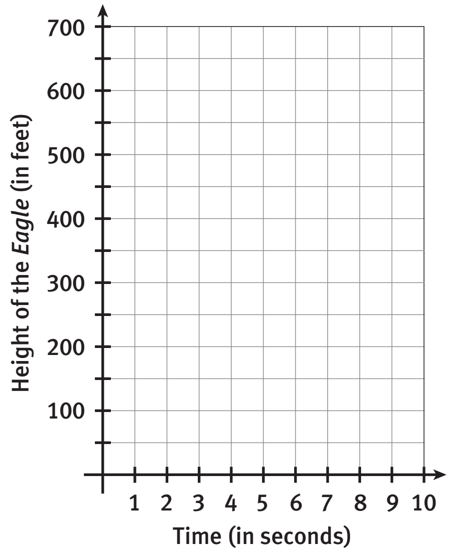
Time Since Parachute Opened (seconds)	0	1	2	3	4	5	6	7	8	9
Height (feet)	625	618	597	562	513	450	373	282	177	58

1. Use the data in the table above. Determine whether the height of the *Eagle* can be modeled by a linear function of time. Explain your reasoning.

SUGGESTED LEARNING STRATEGIES: Create Representations, Close Reading, Activating Prior Knowledge

My Notes

2. Graph the data from the table above Item 1 on the grid below.



3. The height of the *Eagle* can be modeled by the quadratic function $h(t) = kt^2 + h_0$, where k is a constant and h_0 is the initial height of the rocket. You can use the table data to find the values of k and h_0 in the *Eagle's* height function.

a. Solve for the value of h_0 using the point (0, 625). Include the appropriate units for h_0 in your solution.

b. Use a different ordered pair from the table above Item 1 to find the value of k. Write a function for the height of the rocket as a function of time.

READING MATH

A variable with a zero subscript such as h_0 is read "*h* naught," or "*h* sub zero." This means that it is the initial value.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Visualization, Questioning the Text, Create Representations, Think/Pair/Share, Group Presentation

My Notes

4. Use the function from Item 3 to answer these questions.

a. At what time was the rocket's height 450 ft above Earth? Verify that your result agrees with the data in the table.

b. After the parachute opened, how long did it take for the rocket to hit Earth?

Cooper wanted to investigate the flight of a rocket from the time the engine burns out until the rocket lands. He set a device in a second rocket, named *Spirit*, to begin collecting data the moment the engine shut off. Unfortunately, the parachute failed to open. When the rocket begins to descend it will be in free fall.

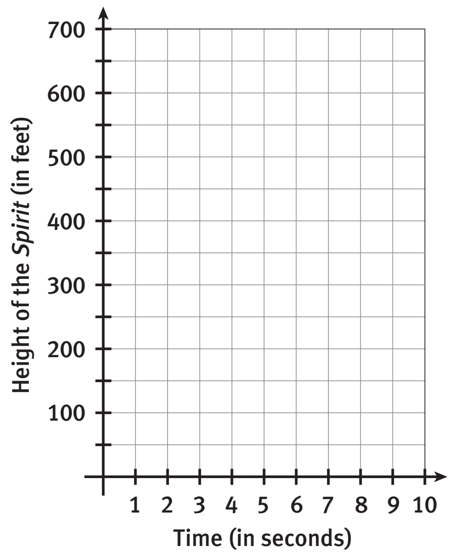
CONNECT TO SCIENCE

A *free falling* object is an object which is falling under the sole influence of gravity.

5. Graph the data for the height of the *Spirit* versus time on the grid.

The *Spirit*

Time Since the Engine Burned Out (s)	Height (ft)
0	512
1	560
2	576
3	560
4	512
5	432
6	320



SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Close Reading, Create Representations

My Notes

6. Use the table and graph in Item 5.

a. How high was the *Spirit* when the engine burned out?

b. How long did it take the rocket to reach its maximum height after the engine cut out?

c. Estimate the time the rocket was in free fall before it reached the earth.

7. Use the table and graph in Item 5.

a. Use a graphing calculator to determine a quadratic $h(t)$ function for the data.

b. Sketch the graph of the function on the grid in Item 5.

8. Use the function found in Item 7 to verify the height of the *Spirit* when the engine burned out.

TECHNOLOGY TIP

For Item 7(a), enter the data from the table in Item 5 into a graphing calculator. Use the calculator's quadratic regression feature to find a representative function.

SUGGESTED LEARNING STRATEGIES: Create Representations, Predict and Confirm, Activating Prior Knowledge, Quickwrite

My Notes

9. Graph the function found in Item 7 on your graphing calculator. Use the graph to approximate the time interval in which the *Spirit* was in free fall. Explain how you determined your answer.

10. The total time that the *Spirit* was in the air after the engine burned out is determined by finding the values of t that makes $h(t) = 0$.

a. Set the equation found in Item 7(a) equal to 0.

b. Completely factor the equation.

c. Identify and use the appropriate property to find the time that the *Spirit* took to strike Earth after the engine burned out.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Create Representations, Think/Pair/Share

My Notes

11. The quadratic formula can also be used to solve the equation from Item 10(a).

a. State the quadratic formula.

b. Use the quadratic formula to determine the total time that the *Spirit* was in the air after the engine burned out. Show your work.

12. Draw a horizontal line on the graph in Item 5 to indicate a height of 544 ft above Earth. Estimate the approximate time(s) that the *Spirit* was 544 ft above Earth.

13. The time(s) that the *Spirit* was 544 ft above Earth can be determined exactly by finding the values of t that make $h(t) = 544$.

a. Set the equation from Item 7(a) equal to 544.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Quickwrite, Create Representations, Think/Pair/Share

My Notes

b. Is the method of factoring effective in solving this equation? Explain your reasoning.

c. Is the quadratic formula effective in solving this problem? Explain your reasoning.

d. Determine the time(s) that the rocket was 544 ft above Earth. Round your answer to the nearest hundredth of a second. Verify that this solution is reasonable compared to the estimated times from the graph, Item 12.

14. Cooper could not see the *Spirit* when it was higher than 528 ft above Earth.

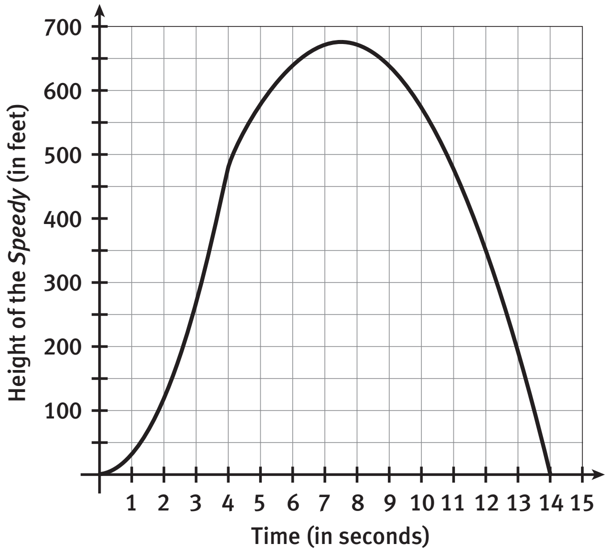
a. Find the values of t for which $h(t) = 528$.

b. Write an inequality for the values of t that are between the two times that the rocket was not within Cooper's sight.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Quickwrite, Visualization, Group Presentation

My Notes

Cooper has a third rocket named *Speedy*. He decided to fire the rocket without a parachute to investigate a rocket's motion in free fall. Cooper represented the launch time as $t = 0$. The graph of the height of the *Speedy* is a piecewise function, shown below.



15. Use the graph above to estimate the answer to each question.

a. At what time did the *Speedy's* engine burn out?

b. What was the maximum height of the rocket and at what time did the rocket reach that height?

c. At what time did the rocket hit Earth?

SUGGESTED LEARNING STRATEGIES: Marking the Text, Simplify the Problem, Create Representations, Think/Pair/Share

My Notes

Cooper is entering the *Speedy* into a contest. The winner is the owner of the rocket that stays above 400 ft for the longest period of time.

16. Draw a horizontal line on the graph above Item 15 to indicate a height of 400 ft above Earth. Estimate when the rocket will be more than 400 ft above Earth.

While *Speedy's* engine is burning for the first 4 seconds, the height is given by the function $h_1(t) = 30t^2$. After the engine burns out, the height is given by $h_2(t) = -224 + 240t - 16t^2$.

17. Write a piecewise function $h(t)$ that expresses the height of the *Speedy* above Earth as a function of time.

18. For the contest, Cooper needs to determine the length of time that the *Speedy* will be 400 ft above Earth. Use the piecewise function for the height of the rocket from Item 17 to determine the exact time(s) that the rocket will be exactly 400 ft above Earth.

a. Explain why two different equations must be solved to determine the time(s) that the rocket will be 400 ft above Earth.

SUGGESTED LEARNING STRATEGIES: Simplify the Problem, Think/Pair/Share, Debriefing

My Notes

b. Write and solve an equation to determine the time that the rocket will be 400 ft above Earth during the time interval for which $0 \le t \le 4$.

c. Write and solve an equation to determine the time that the rocket will be 400 ft above Earth during the time interval for which $t > 4$.

d. Determine the length of time that the rocket will be 400 ft above Earth.

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

The equation $h(t) = -13t^2 + 130t + 312$ represents the flight of a model rocket.

1. Determine when the rocket hits Earth.
2. Graph the equation. Determine the time at which the rocket reaches its maximum height.
3. Determine the times at which the rocket is higher than 423 ft. Explain how you arrived at your solution.

A model rocket burns for 3.5 s. The rocket will be 286 ft in the air when the engine burns out. After the rocket's engine burns out, the rocket's height is given by the function $h(t) = 286 + 190t - 16t^2$.

4. Determine the total time after the rocket is launched that it will be in the air.
5. Determine the times after the rocket is launched that it will be 450 ft in the air.
6. MATHEMATICAL REFLECTION Why are quadratic functions used to model free-fall motion instead of linear functions?

Solving Quadratic Equations

EGG DROP

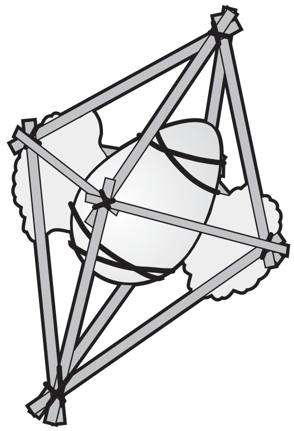
Embedded Assessment 2

Use after Activity 5.5.

Every fall the Physics Club hosts an annual egg drop contest. The goal of the egg-drop contest is to construct an egg-protecting package capable of providing a safe landing upon falling from a fifth-floor window.

During the egg drop contest, each contestant drops an egg about 64 ft to a target placed at the foot of a building. The target is laid on the cement with dimensions of about 10 ft square. Points are given for targeting, egg survival, and time to reach the target.

Colin wanted to win the egg drop contest, so he tested one of his models with three different ways of dropping the package. These equations represent each method.

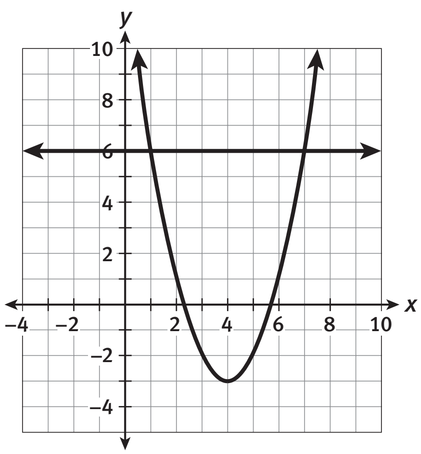


Method A	$h(t) = -16t^2 + 64$
Method B	$h(t) = -16t^2 - 8t + 64$
Method C	$h(t) = -16t^2 - 48t + 64$

1. You can solve quadratic equations by graphing, factoring, using square roots or the quadratic formula. Solve the three equations above. Use a different solution method for each equation to find t when $h(t) = 0$. Show your work, and explain your reasoning for choosing the method you did.

2. Colin found that the egg would not break if it took longer than 1.5 seconds to hit the ground. Which method(s) will not result in the egg breaking?

3. Find the solutions to the quadratic equation $x^2 - 8x + 13 = 6$ using the graph below. Explain.



Solving Quadratic Equations

EGG DROP

	Exemplary	Proficient	Emerging
Math Knowledge 1, 3	The student: • Uses different methods to solve all three equations correctly. (1) • Finds the correct solutions to the quadratic equation. (3)	The student: • Uses different methods to solve only two of the equations correctly. (1) OR • Uses the same method to solve the three equations correctly. (1) • Finds only one of the correct solutions. (3)	The student: • Solves only one of the equations correctly. (1) • Finds neither correct solution. (3)
Problem Solving 2	The student determines for which of the three methods the egg will not break. (2)	The student makes a correct determination for two of the methods.	The student makes a correct determination for only one of the methods.
Communication 1	The student gives complete and coherent reasons for choosing each method of solution. (1)	The student gives reasons for choosing two of the methods of solving the equations.	The student gives a reason for choosing only one method of solution.

ACTIVITY 5.1

Identify whether each function is quadratic. Explain.

1. $y = 2x - 3^2$

2. $y = 3x^2 - 2x$

3. $y = 2 - \frac{3}{x^2} + x$

Write each quadratic function in standard form.

4. $y = 3x - 5 + x^2$

5. $y = 6 - 5x^2$

6. $y = -0.5x + \frac{3}{4}x^2 - \pi$

For each function, complete the table of values. Graph the function and identify the maximum or minimum of the function.

7. $y = x^2 + 4x - 1$

x	*y*
0	
−1	
−2	
−3	
−4	

8. $y = -x^2 + 8x - 13$

x	*y*
2	
3	
4	
5	
6	

ACTIVITY 5.2

To obtain each function, identify the transformations that need to be applied to the graph of the parent function $y = x^2$. Graph each function.

9. $y = \frac{1}{3}x^2 + 2$

10. $y = 2x^2 - 6$

11. $y = -3x^2 + 2$

12. $y = -x^2 + 1$

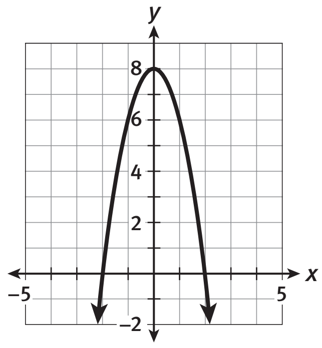
Write the equation of the function described by each transformation of the graph of $y = x^2$.

13. translated down 4 units

14. shrunk vertically by a factor of $\frac{4}{5}$

15. reflected over the x-axis, stretched vertically by a factor of 2, and translated down 5 units

16. Write an equation for the function displayed.



Practice

ACTIVITY 5.3

A cliff diver in Acapulco jumps. His height *y* meters as a function of his distance from the cliff base, *x* meters, is given by the quadratic function $y = 100 - x^2$ for $x \geq 0$.

17. Graph the cliff diver's height equation.

18. What are the solutions to the equation $100 - x^2 = 0$?

19. How do the solution to the equation in Item 18 compare to the *x*-intercepts of the graph in Item 17?

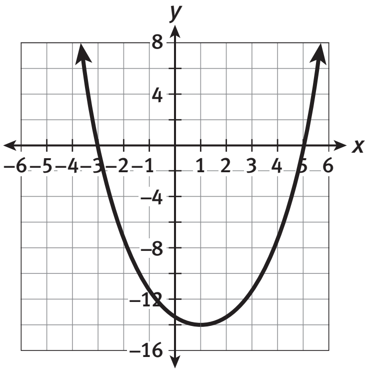
20. How high is the cliff from which the diver jumps?

21. How far from the base of the cliff does the diver hit the water?

22. What are the solutions to the equation $x^2 - 6x = -5$?

a. $-1, 5$
b. $2, 3$
c. $1, 5$
d. $-2, 3$

23. What is the equation of the parabola graphed below?



a. $y = x^2 - 2x - 15$
b. $y = x^2 + 2x - 15$
c. $y = x^2 - 8x + 15$
d. $y = x^2 + 8x + 15$

ACTIVITY 5.4

24. Solve: $(x - 1)^2 = 18$

25. Solve: $x^2 - 10x + 25 = 6$

26. Given the equation $x^2 - 6x = 3$, what number should be added to both sides to complete the square?

a. -3
b. 6
c. 9
d. 36

27. What are the solutions of $5x = x^2 + 2$?

a. $\frac{-5 \pm \sqrt{17}}{2}$
b. $\frac{5 \pm \sqrt{17}}{2}$
c. $\frac{-5 \pm \sqrt{33}}{2}$
d. $\frac{5 \pm \sqrt{33}}{2}$

28. How many real solutions does the equation $4x^2 + 4x + 1 = 0$ have?

a. none
b. one
c. two
d. more than two

ACTIVITY 5.5

For Items 29–31, use the equation $h(t) = -9t^2 + 72t + 297$, which represents the flight of a model rocket.

29. Determine when the rocket will hit the ground.

30. Graph the equation and determine the time at which it reaches its maximum height.

31. Determine the times at which the rocket is higher than 320 ft. Explain how you arrived at your solution.

32. Which of the functions could represent the path of a model rocket? Explain your reasoning.

a. $h(t) = 15 + 3(t - 8)$
b. $h(t) = |5t - 4| + 320$
c. $h(t) = 421 + 11t - 24t^2$
d. $h(t) = 15t^2 - 125t + 229$

33. Create an equation that could model the height of a model rocket as a function of time. Determine when your rocket would hit the ground, and find the maximum height of your rocket.

Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking on the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.

- How are quadratic functions used to model, analyze, and interpret mathematical relationships?
- Why is it advantageous to know a variety of ways to solve and graph quadratic functions?

Academic Vocabulary

2. Look at the following academic vocabulary words:

- parabola
- parent function
- quadratic formula
- quadratic function
- real roots of an equation
- transformation
- vertex of a parabola

Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessments in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

Unit Concepts	Is Your Understanding Strong (S) or Weak (W)?
Concept 1	
Concept 2	
Concept 3	

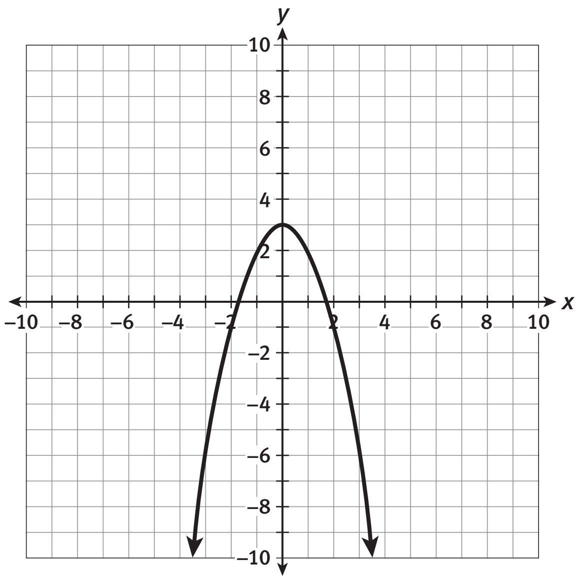
a. What will you do to address each weakness?

b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?

1. Which equation represents the graph shown below?

1. Ⓐ Ⓑ Ⓒ Ⓓ



A. $y = x^2$

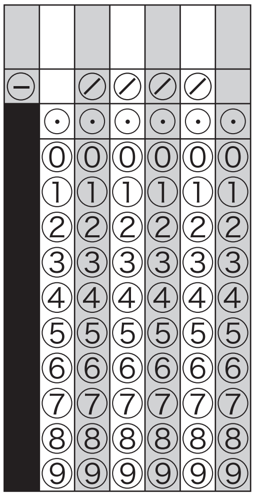
B. $y = -x^2$

C. $y = -x^2 + 2$

D. $y = -x^2 + 3$

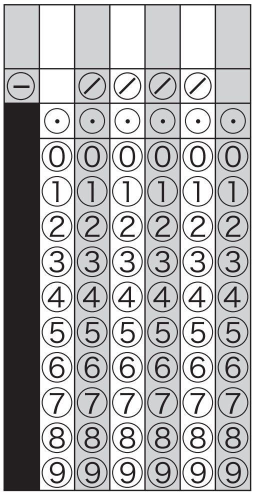
2. A baseball player hits a ball to the outfield according to the following function $h(t) = -16t^2 + 48t$, where h is the height of the ball (in feet) and t is the numbers of seconds after the ball is hit. At what time t, in seconds, does the ball hit the ground after being hit?

2.



3. The length of a rectangle is 4 meters more than its width. The area is 12 square meters. What is the width, in meters, of the rectangle?

3.



Read
Solve
Explain

4. Given this equation: $y + 7 = (x - 4)^2$

Part A: Find the vertex and the axis of symmetry.

Answer and Explain

Part B: Write the equation in vertex form:

Answer and Explain